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Optimal product positioning based on paired comparison data

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Abstract

A new approach for analyzing paired comparison data is proposed which combines a probabilistic ideal point model with product positioning issues. Unlike traditional approaches based on paired comparison data the same formulation is used for estimating a joint space representation of consumer segments and products as well as for determining optimal (new) product positioning options in a relevant product-market. A Monte Carlo experiment is presented and real-world coffee market data are used to show advantages of the new approach. © 1999 Elsevier Science S.A. All rights reserved.

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1. Introduction

Product positioning and product design are generally viewed as closely related important problems in marketing research that deal with the generation of 'promising' options for a firm that plans to extend or modify its existing product lines, see, e.g., Schmalensee and Thisse (1988), Green and Krieger (1989), Kaul and Rao (1995) for reviews. 'Promising' is typically operationalized using objectives like, e.g., (expected) additional sales volume, market share or profit for the firm that would arise if a (new) product could be positioned or designed according to the generated option(s). A distinction between positioning options

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and design options can be made in so far as positioning options are described using (perceptual) product attributes that build a basis for consumer's product choice decisions (e.g., cheapness, printing quality, or speediness of a laser printer) whereas design options are stated in terms of the underlying (physical) product characteristics (e.g., price, linewidth or number of pages per minute).

In the conceptual framework for the generation of such promising options, as introduced by Shocker and Srinivasan (1974) and extended, e.g., by Kaul and Rao (1995), four principal stages are distinguished: (i) the definition of the relevant product-market, (ii) the description of the relevant product-market using (important) product attributes and/or product characteristics, (iii) the modeling of consumer's product choice decisions via data collection and parameter estimation, and (iv) the generation of positioning and/or design options via choice simulation and optimization. Various approaches have been proposed in which different methods at each of these stages are applied. However, as the modeling assumptions across the principal stages typically vary within an approach, the generated positioning and/or design options suffer from suboptimality. This is especially true for approaches where paired comparisons form the major data collection method. There, in the third stage, e.g., LINMAP is used for deriving joint space representations where it is assumed that alternatives with higher (constant) utility values in the pairs are preferred. Then, in the fourth stage, search algorithms for generating optimal (new) product positioning options are applied to these joint space representations that rely on the assumption that the selection of an alternative from a set of alternatives is proportional to some share of (constant) utility values, see, e.g., Shocker and Srinivasan (1974), Sudharshan et al. (1987) or Sudharshan et al. (1995).

Our concern in this paper is to present a new approach for optimal product positioning by analyzing paired comparison data where the assumptions for modeling consumer's product choice decisions in the third stage and for generating product positioning options in the fourth stage are the same. In the next section we start with a more detailed description of product positioning and design issues as basis for the understanding of our approach. After the mathematical formulation of our approach in Section 3 we present a Monte Carlo experiment in Section 4 to show how the new approach behaves with respect to various performance measures in comparison with a counterpart selected from known approaches. In an empirical example in Section 5 coffee market data are analyzed in order to demonstrate additional advantages of the proposed approach.

2. Principal stages of product positioning and design

The description of the first two of the just mentioned four principal product positioning and design stages is kept to a minimum while for the last two which are of particular interest for the integration of our new approach into this area, additionally, overviews on models and techniques used in these stages are provided.

(i) In the first stage, the relevant product market which comprises products and target segments of the potential competitive environment is identified. Depending on the objective pursued, different databases and methods for market definition can be used, see, e.g., Urban and Hauser (1993), (pp. 102) who discuss approaches based on segmentation and on product substitution.

(ii) In the second stage, important (perceptual) product attributes and/or product characteristics have to be determined that allow a description of the relevant product market and the corresponding positionings (attribute-level combinations) and/or designs (characteristic-level combinations) of the existing products. Standard techniques for deriving the current product positionings consist in collecting, e.g., property ratings and/or similarity ratings from respondents and deriving low-dimensional attribute spaces with point representations for the existing products, see, e.g., Shocker and Srinivasan (1979) or Huber and Holbrook (1979) for comparisons of different techniques which use, e.g., (non)metric scaling, factor analysis, or discriminant analysis.

(iii) In the third stage, consumer's product choice decisions are integrated in the modeling efforts. Based on Brunswick (1952)'s 'lens' model, it is assumed that consumers condense (physical) product characteristics and firms' marketing mix efforts to few (perceptual) product attributes on which their product preferences and (assuming utility maximization as choice principle) their product choice decisions essentially depend. Therefore, preferential and/or choice data from respondents are analyzed in order to relate product characteristics via product attributes and preferences to consumer's product choice decisions in the product market under view, see, e.g., Kaul and Rao (1995) for a more detailed discussion. Mainly two different measurement and modeling approaches for this purpose can be distinguished: MDS (multidimensional scaling)-based approaches and CA (conjoint analysis)-based approaches.

The usual procedure in MDS-based approaches is the following: Products are represented by product points and consumer segments by the so-called ideal points in the low-dimensional attribute space. Inverse (or negative) distances between ideal points and product points are assumed to be related to (constant) product utility values. A product whose point has a smallest distance (with respect to a set of competing products) to a segment-specific ideal point is selected by the corresponding segment – as a consequence of utility maximization as choice principle. For obtaining these so-called joint space representations often paired comparison data are collected through repeatedly exposing respondents to pairs of products and asking them to select the most preferred product in the pair (binary paired comparisons as choice data) or to judge the superiority of the most preferred product in the pair (graded paired comparisons or constant-sum comparisons as judgmental data). Paired comparison experiments have advantages since this kind of data collection convinces through task simplicity for the respondents and the ability to capture variation in consumer's product choice decisions, see, e.g., Dillon et al. (1993) for a discussion of advantages and applications to marketing research.

Then, clustering methods and ideal point models are applied for estimating the joint space parameters. Table 1 provides an overview on ideal point models for this purpose. The models differ with respect to the analyzed data types, constraints (where 'external' means that product positionings in the attribute space are known from the second stage just mentioned whereas 'constrained' implies that product positionings depend on known product characteristics according to some functional form) and the estimation algorithms applied. Newer ideal point models differ from older ones in so far as the allocation of respondents to segments and the estimation of the segment-specific joint space parameters is performed simultaneously (e.g., Böckenholt and Gaul, 1989; DeSoete, 1990; Böckenholt and Böckenholt, 1991; Baier and Gaul, 1996; Wedel and DeSarbo, 1996) and that product utilities are stochastically modeled assuming either normally distributed segment-specific ideal points (e.g., Böckenholt and Gaul, 1986; DeSoete et al., 1986; Kamakura and Srivastava, 1986; DeSoete, 1990; Baier and Gaul, 1996) or constant utility values superimposed by additive random error (e.g., Cooper and Nakanishi, 1983; DeSarbo et al., 1987; DeSarbo et al., 1987; DeSarbo and Hoffman, 1987; Böckenholt and Gaul, 1988, 1989; Böckenholt and Böckenholt, 1991; Wedel and DeSarbo, 1993).

It should be noted that from the models in Table 1 only those for analyzing pick any/*n* or pick 1/*n* data provide a straightforward possibility for predicting choices among larger sets of alternatives, i.e. market shares for existing or new products. The modeling assumptions used during joint space estimation based on paired comparisons (i.e., the 'better' product from a pair of products is always selected) can be extended to the pick 1/*n* context (i.e., the 'best' product from a set of products is always selected). This assumption is called the 'deterministic' choice rule. More often, some share of weighted (constant) utility values (the so-called 'probabilistic' choice rule which contains the 'deterministic' choice rule as a special case) has been used. Especially, for product markets with frequently purchased products and/or for preference modelings at the segment level the 'probabilistic' choice rule has proven to be more realistic, see, e.g., Shocker and Srinivasan, 1979; Sudharshan et al. (1988) and the references therein.

The usual procedure in CA-based approaches differs from the above described MDS situation in so far as typically the distinction between product attributes and product characteristics is neglected in favor of a one-to-one relation, see, e.g., Kaul and Rao (1995) for stressing this point, and that respondents are asked to evaluate hypothetical products or stimuli and not the existing products as in MDS-based approaches. The relevant attributes and the levels of the attributes are restricted to small numbers so that full or fractional factorial designs can be used for a systematic construction of not too large stimulus sets

Table 1
Overview on ideal point models for external and/or constrained analyses of judgmental or choice data

Description of models	Data types	Constraints	Algorithms applied	References
Linear progr. techniques for multidim. analysis of preferences	Paired comp. data	External	Linear progr.	Srinivasan and Shocker (1973) (LINMAP)
Logit model for external preference analysis	Paired comp. data	External	ML, GLS	Cooper and Nakanishi (1983)
Prob. ideal point model; wandering ideal point model	Paired comp. data; graded paired comp. data	Internal; external	ML, nonlinear progr.	Böckenholt and Gaul (1986); DeSoete et al. (1986)
Generalized unfolding model	Rank-order data; rating data	External; constr.	ALS, nonlinear progr.	DeSarbo and Rao (1986) (GENFOLD2)
Ideal point prob. choice model	Pick $1/n$ data; rank-order data	External	ML, nonlinear progr.	Kamakura and Srivastava (1986)
Prob. unfolding MDS model	Paired comp. data	Internal; external; constr.	ML, nonlinear progr.	DeSarbo et al. (1987), DeSarbo et al. (1987)
Multidim. unfolding threshold model	Pick any/n data	External; constr.	ML, nonlinear progr.	DeSarbo and Hoffman (1987)
Latent class prob. ideal point model	Pick any/n data; rank-order data	Internal; external	ML, nonlinear progr., EM-alg.	Böckenholt (1989), Böckenholt and Gaul (1989)
Latent class wandering ideal point model	Paired comp. data	Internal; external	ML, nonlinear progr., EM-alg.	DeSoete (1990)
Latent class unfolding model	Pick any/n data	Internal; external; constr.	ML, nonlinear progr., EM-alg.	Böckenholt and Böckenholt (1991)
Simultaneous prob. ideal point model	Paired comp. data	Internal; external; constr.	ML, nonlinear progr., CML-alg.	Baier and Gaul (1996)
Simultaneous segmentation and product positioning	Rank-order data; rating data	Internal; external; constr.	ML, nonlinear progr., EM-alg.	Wedel and DeSarbo (1996) (LCSTUN)

(ALS = alternating least squares, CML = classification maximum likelihood, EM = expectation maximization, GLS = generalized least squares, ML = maximum likelihood, Alg. = algorithm, Comp. = comparisons, Constr. = constrained, Multidim. = Multidimentional, Prob. = probabilistic, Progr. = programming) for data collection. As in the MDS context, paired comparison experiments belong to the standard data collection methods in CA-based approaches, see, e.g., Wittink et al. (1994) for a recent survey on commercial applications of conjoint analysis in Europe, where the paired comparison-based ACA procedure was the most popular one.

The observed response data are analyzed using cluster analysis and regression-like estimation algorithms at the individual or segment level. Table 2

Table 2
Overview on CA-based estimation techniques for analyzing judgmental or choice data

Description of models	Data types	Constraints	Algorithms applied	References
Nonmetric CA	Rank-order data	External	ALS	Kruskal (1965) (MONANOVA)
Linear progr. techniques for multidim. analysis of preferences	Paired comp. data	External	Linear progr.	Srinivasan and Shocker (1973) (LINMAP)
Metric CA	Rating data	External	OLS	Carmone et al. (1978)
Logit analysis; choice-based CA	Pick $1/n$ data	External	ML, nonlinear progr.	Louviere and Woodworth (1983), Sawtooth (1993)
Hybrid CA analysis	Rank-order data; rating data	External	OLS	Green (1984)
Adaptive CA	Rating data, paired comp. data	External	OLS	Johnson (1987), Sawtooth (1994) (ACA)
Hierarchical benefit segmentation	Rating data	External	ALS	Kamakura (1988)
Clusterwise regression	Rating data	External	ALS	Wedel and Kistemaker (1989)
Fuzzy clusterwise regression	Rating data	External	ALS	Wedel and Steenkamp (1989)
Latent class CA	Paired comp. data	External	ML, nonlinear progr., EM-alg.	DeSoete (1990)
Latent class metric CA	Rating data	External	ML, nonlinear progr., EM-alg.	DeSarbo et al. (1992)
Concomitant variable latent class CA	Rank-order data	External	ML, nonlinear progr., EM-alg.	Kamakura et al. (1994)
Latent class choice-based CA	Pick $1/n$ data	External	ML, nonlinear progr., EM-alg.	DeSarbo et al. (1995)

(ALS = alternating least squares, EM = expectation maximization, ML = maximum likelihood, OLS = ordinary least squares, Alg. = algorithm, Comp. = comparisons, Multidim. = multidimensional, Progr. = programming) provides an overview on respective techniques. As with the ideal point models within the MDS-based approaches, newer estimation algorithms can be distinguished from older ones through their ability to determine the allocation of respondents to segments and segment-specific model parameters simultaneously, see, e.g., Kamakura (1988), Wedel and Kistemaker (1989), Wedel and Steenkamp (1989), DeSoete (1990), DeSarbo et al. (1992), DeSarbo et al. (1995) and Kamakura et al. (1994). The resulting segment-specific partworth functions or estimated preferences for attribute-levels can then be used to predict (constant) utility values for the existing products through adding the respective partworth values. For predictions of shares of choices similar rules as described for the MDS-based approaches can be used.

The modeling of consumer's product choice decisions developed so far also provides a basis for the evaluation of attribute-level combinations (positioning options) for new products. For this purpose, (constant) utilities are calculated in the same way as for existing products and (simulated) shares of choices for the new product are estimated using the already mentioned 'deterministic' or 'probabilistic' choice rules.

More cumbersome from a practical point of view is the evaluation of positioning options with respect to profit and/or the evaluation of design options with respect to share of choices, sales volume, market share or profit, since in these cases the relations between marketing variables and product characteristics (that build up main parts of the product's variable costs) and the positionings in the attribute space (that make up consumer's product choice decisions) must be known. Here, especially in more technical product categories, sometimes linear relations between product attributes and product characteristics are assumed. Given that the number of product characteristics is not too large, regression analysis can be used to estimate the transformation coefficients, e.g., Narasimhan and Sen (1989) demonstrated in an application that (perceptual) attributes like copy quality in copiers are linearly related to (physical) characteristics like linewidth, darkness, gloss, spots, or streaks. Another possibility can be an application of constrained ideal point models as listed in Table 1 where the product coordinates in the attribute space are linear combinations of product characteristics. However, if the number of product characteristics is too large, a regression-like estimation of the transformation coefficients seems not to be appropriate. In such cases, research concerning methods incorporating engineering experience and experimental studies is still going on, see, e.g., Hauser and Clausing (1988), Akao (1990), and Gustafsson (1996). Remember that these problems are less valid for CA-based approaches, since here (as mentioned before) typically a one-to-one relation between product characteristics and product attributes exists. Carroll (1997) assumes that this 'advantage' from a practical point of view is one of the major reasons why CA-based approaches are currently more popular than the MDS-based counterparts.

Table 3
Summary on optimal product positioning techniques

Description of objectives	Number of products	Choice rules	Solution principles	Algorithms applied	References
Techniques in MDS-based approaches					
Share of choices; market share	Single product	Det.; prob.	Heuristic	Grid search; nonlinear progr.	Shocker and Srinivasan (1974)
Share of choices	Product line	Det.	Exact	Geometric approach	Albers (1977) (SILOP)
Share of choices; market share	Single product	Det.	Exact	Geometric approach	Albers and Brockhoff (1979) (PROPOSAS)
Share of choices; market share	Single product	Det.	Exact	Geometric approach	Zufryden (1979) (ZIPMAP)
Profit	Single product	Det.; prob.	Heuristic; exact	Nonlinear progr.; branch & bound	Bachem and Simon (1981)
Share of choices	Single product	Det.	Heuristic	Geometric approach	Gavish et al. (1983)
Share of choices; market share	Single product	Det.; prob.	Heuristic	Nonlinear progr.	Sudharshan et al. (1987) (PRODSRCH)
Share of choices; market share	Product line	Det.; prob.	Heuristic	→ PRODSRCH (modified)	Sudharshan et al. (1988) (DIFFSTRAT)
Share of choices; profit	Single product	Det.; prob.	Exact	→ PROPOSAS (modified)	Albers (1989)
Profit	Single product	Det.; prob.	Exact	Game-theoretic approach	Choi et al. (1990, 1992)
Profit	Single product	Det.; prob.	Exact	Game-theoretic approach	Horsky and Nelson (1992)
Share of choices; sales volume	Single product	Det.; prob.	Heuristic	→ PRODSRCH (modified)	Sudharshan et al. (1995) (NICHER)
Techniques in CA-based approaches					
Sales volume	Single product	Det.; prob.	Heuristic	Integer progr.	Zufryden (1979)
Sales volume; market share; profit	Single product	Det.; prob.	Exact; heuristic	Enumeration; branch & bound; nonlinear progr.	Green et al. (1981) (POSSE)
Profit	Product line	Det.	Heuristic	Greedy;linear progr. (relaxation)	Green and Krieger (1985) (DESOP)
Share of choices	Single product	Det.	Heuristic	Dynamic progr. heuristic	Kohli and Krishnamurti (1987)

Table 3. *Continued*

Description of objectives	Number of products	Choice rules	Solution principles	Algorithms applied	References
Welfare; profit	Product line	Det.	Heuristic	Reverse-greedy	Dobson and Kalish (1988)
Welfare; sales volume; profit	Product line	Det.	Exact	Linear progr.; branch & bound	McBride and Zufryden (1988)
Share of choices; profit	Product line	Det.	Heuristic	Dynamic progr. heuristic	Kohli and Sukumar (1990)
Market share; profit	Product line	Prob.	Heuristic	Divide-and-conquer	Green and Krieger (1992) (SIMOPT)
Profit	Single product	Det.; prob	Exact	Game-theoretic approach	Choi and DeSarbo (1993, 1994)
Profit	Product line	Det.	Heuristic	Greedy	Dobson and Kalish (1993)
Profit	Product line	Prob.	Heuristic	Advanced greedy	Gaul et al. (1995) (PROLIN)
Share of choices	Single product	Det.	Heuristic	Genetic algorithms	Balakrishnan and Jacob (1996)

(Det. = deterministic, Prob. = probabilistic, Progr. = programming)

(iv) In the fourth stage, the product choice model developed in the previous stage is used to generate product positioning and/or design options that maximize some prespecified objectives. If, e.g., the generation of a positioning option for a new product is desired that maximizes the new product's share of choices, a position that maximizes the simulated share of choices has to be found. Various techniques have been proposed for this purpose. Table 3 provides a summary on these efforts. The different techniques are distinguished with respect to objectives, number of products, choice rules, solution principles, and the algorithms applied. We can see that Shocker and Srinivasan (1974)'s early formulation of the problem from an MDS-based perspective was a starting point for other researchers to develop enumerative or heuristic techniques concerning optimal positionings for single products or product lines. Some newer approaches incorporate game-theoretic considerations for modeling competitive reactions with respect to sets of (established) products. Concerning CA-based approaches, early formulations of this problem were given by Zufryden (1979) and Green et al. (1981), newer techniques focus on computational problems to determine optimal positionings for single products or for product lines. As with the MDS-based approaches, some newer methods use game-theoretic formulations to model competitive reactions.

One should keep in mind that in both CA- and MDS-based approaches the estimation of the preference model parameters (e.g., the ideal point coordinates within MDS-resp. the partworth functions within CA-based approaches) and the prediction of the shares of choices for existing and for new products rely on different modeling assumptions when paired comparison data are used. In the Monte Carlo experiment later on, we will show how this usage of different modeling assumptions influences the accuracy of the choice predictions compared to our new approach, for which the same model formulation is used for parameter estimation and for optimal positioning. In the following, this approach, in which Baier and Gaul (1996)'s paired comparison based simultaneous probabilistic ideal point model formulation is generalized in order to predict shares of choices among sets of products (though retaining paired comparisons as data collection method), is described.

3. New approach

3.1. Model formulation

We represent a set S of products by points in an r-dimensional space with deterministic coordinate vectors $\boldsymbol{x}_j = (x_{j1}, \ldots, x_{jr})'$, $j \in S$, determined e.g. by preliminary analyses of perceptual data in the first two product positioning and design stages of the conceptual framework.

In the same space, we describe T consumer segments by ideal points with stochastic coordinate vectors $\boldsymbol{v}_t = (v_{t1}, \ldots, v_{tr})'$ $(t = 1, \ldots, T)$ which follow multivariate normal distributions with mean $\boldsymbol{\mu}_t = (\mu_{t1}, \ldots, \mu_{tr})'$ and covariance matrix $\boldsymbol{\Sigma}_t = ((\sigma_{tpp'}))_{r \times r}$.

As in traditional approaches for product positioning and design we assume that consumers are allocated to consumer segments, that inverse distances between segment-specific ideal points and product points reflect segment-specific product utilities, and (as a consequence of utility maximization as choice principle) that smallest distance with respect to a set of competing products implies choice. Additionally, it is assumed that consumer's product choice decisions vary across consumers, segments and product choice situations and that consumers sample an ideal point from their corresponding segment-specific ideal point distribution in each concrete choice situation. Consequently, we use the notation

$$R_{j|S} = \{z \in \mathbb{R}^r \mid (z - \boldsymbol{x}_j)'(z - \boldsymbol{x}_j) \leqslant (z - \boldsymbol{x}_k)'(z - \boldsymbol{x}_k)\ \forall k \in S\} \tag{1}$$

for what we have called preference subset of product j (which contains all points for which j is the closest product with respect to S) and calculate the probability that consumers from segment t prefer product j to any other product from S by

$$p_{tj|S} = \Pr(v_t \in R_{j|S}) = \int_{z \in R_{j|S}} f_t(z)\,dz \tag{2}$$

with

$$f_t(z) = \frac{1}{\sqrt{(2\pi)^r \det(\Sigma_t)}} \exp(-\tfrac{1}{2}(z-\mu_t)'\Sigma_t^{-1}(z-\mu_t)). \tag{3}$$

Using λ_t as the relative size of segment t ($\sum_{t=1}^{T}\lambda_t = 1$) we get

$$p_{j|S} = \sum_{t=1}^{T} \lambda_t p_{tj|S} \tag{4}$$

as overall share of choices for product j.

Note that Eq. (2) generalizes well-known formulations from paired comparison-based probabilistic ideal point models, see, e.g., Böckenholt and Gaul (1986), DeSoete et al. (1986), DeSoete (1990), and Baier and Gaul (1996). In the case $|S| = 2$ a closed-form solution of the probability

$$p_{tj|\{j,k\}} = \Pr(v_t \in R_{j|\{j,k\}}) = \Phi\left(\frac{x_k'x_k - x_j'x_j - 2(x_k - x_j)'\mu_t}{\sqrt{4\ (x_k - x_j)'\Sigma_t\ (x_k - x_j)}}\right) \tag{5}$$

that product j is preferred to product k is possible. (Φ denotes the standard normal distribution.)

For $|S| > 2$ an analytical solution for the probability expression (2) is not known, but hypercube approximations – where the centroid of a hypercube indicates whether all points of the hypercube are assumed to belong to $R_{j|S}$ or not – and approximations of the multivariate normal distribution can be used for evaluations, see, e.g., Gupta (1963) and Krishnaiah (1980). The hypercube approximation is described in more detail in the appendix, see also Kamakura and Srivastava (1986)'s application of Clark's rule in their (unsegmented) ideal point probabilistic choice model formulation for analyzing pick $1/n$ data as an alternative possibility for approximation.

3.2. Data collection and parameter estimation

Now, the data collection and parameter estimation part of the new approach can be described as follows:

We use for the tth segment-specific ideal point a multivariate normal distribution with parameter vector $\theta_t = (\mu_{t1}, \ldots, \mu_{tr}, \sigma_{t11}, \sigma_{t12}, \ldots, \sigma_{trr})'$. For estimating the vector of ideal point parameters $\theta = (\theta'_1, \ldots, \theta'_T)'$, paired comparisons are collected from N respondents. As a result we get – with i as an index for respondents and j, k as indices for products – a data array $\boldsymbol{Y}$ with elements

$$y_{ijk} = \begin{cases} 1 & \text{if respondent } i \text{ prefers product } j \text{ to product } k, \\ 0 & \text{otherwise,} \end{cases} \quad \forall i, j, k. \tag{6}$$

Note that this notation allows for missing values in the data. As segment-specific model parameters are needed, an additional, possibly unknown non-overlapping segmentation matrix $\boldsymbol{H}$ with elements

$$h_{ti} = \begin{cases} 1 & \text{if respondent } i \text{ belongs to segment } t, \\ 0 & \text{otherwise,} \end{cases} \quad \forall t, i \tag{7}$$

is introduced from which we get the relative segment sizes $\lambda_t = \sum_{i=1}^{N} h_{ti}/N$.

For obtaining estimates of θ and $\boldsymbol{H}$, sequential techniques, where the respondents are a priori assigned to segments by clustering methods (e.g., by Ward- or k-means-clustering based on the paired comparison data) before θ is computed, and simultaneous techniques, where θ and $\boldsymbol{H}$ are jointly determined, are possible.

A computationally tractable example for a more advanced simultaneous technique is based on the classification maximum likelihood method, see, e.g., Bryant and Williamson (1978, 1986). In this context, estimates of θ and $\boldsymbol{H}$ are obtained by minimizing the negative log-likelihood function

$$-\ln L(\theta, \boldsymbol{H}|\boldsymbol{Y}) = -\sum_{t=1}^{T} \sum_{j \in S} \sum_{k \in S \setminus \{j\}} n_{tjk} \ln(p_{tj|\{j,k\}}) \tag{8}$$

with

$$n_{tjk} = \sum_{i=1}^{N} h_{ti} y_{ijk}, \quad \forall t, j, k, \quad h_{ti} \in \{0,1\}, \quad \forall t, i, \quad \sum_{t=1}^{T} h_{ti} = 1, \forall i. \tag{9}$$

Here, n_{tjk} denotes the number of respondents in segment t who prefer product j to product k. Note that in Eq. (8) an independent sampling of ideal point coordinates across segments, respondents, and choice situations is assumed as in other papers that derive maximum likelihood estimates from paired comparison data, see, e.g., Böckenholt and Gaul (1986), DeSoete et al. (1986), DeSoete (1990), and Baier and Gaul (1996). Since

$$
\begin{aligned}
-\ln L(\boldsymbol{\theta}, \boldsymbol{H} | \boldsymbol{Y}) &= -\sum_{t=1}^{T} \sum_{j \in S} \sum_{k \in S \setminus \{j\}} \left(\sum_{i=1}^{N} h_{ti} y_{ijk} \right) \ln(p_{tj|\{j,k\}}) \\
&= -\sum_{i=1}^{N} \sum_{t=1}^{T} h_{ti} \underbrace{\sum_{j \in S} \sum_{k \in S \setminus \{j\}} y_{ijk} \ln(p_{tj|\{j,k\}})}_{=: L_{it}(\boldsymbol{\theta}_t | \mathbf{Y})},
\end{aligned}
\tag{10}
$$

the estimation of the model parameters can be simplified according to the algorithm shown in Table 4, consisting of an initialization phase and a two-step iteration phase where the actual estimates of $\boldsymbol{\theta}$ and $\boldsymbol{H}$ are alternatingly improved. Step 1 of the algorithm's iteration phase is similar to the estimation method of Böckenholt and Gaul (1986)'s and DeSoete et al. (1986)'s probabilistic ideal point model, where for a given segmentation matrix $\boldsymbol{H}$, estimates of $\boldsymbol{\theta}$ are obtained. In this case we denote the log-likelihood function by $\ln L(\boldsymbol{\theta}|\boldsymbol{H}, \boldsymbol{Y})$. In step 2, $\boldsymbol{H}$ is improved by allocating respondents to segments in such a way that Eq. (10) is minimized. Fifty restarts are used to consider local optima.

For model selection, values of AIC (Akaike Information Criterion, see Akaike, 1977) of the type

$$
\mathrm{AIC} = -2 \ln L(\hat{\boldsymbol{\theta}}, \boldsymbol{H} | \boldsymbol{Y}) + 2\mathrm{NEP}, \tag{11}
$$

Table 4
Algorithm for parameter estimation

{Initialization phase:}
Set $l := 0$. Choose an arbitrary (non-overlapping) segmentation matrix $\boldsymbol{H}^{(0)}$, initial estimates $\boldsymbol{\theta}^{(0)}$ of the vector of ideal point parameters, and a small $\delta > 0$.

{Iteration phase:}

Repeat	{Step 1 (Estimation):} Set $l := l + 1$. Compute a new estimate $\boldsymbol{\theta}^{(l)}$ of $\boldsymbol{\theta}$ by minimizing $-\ln L(\boldsymbol{\theta}\|\boldsymbol{H}^{(l-1)}, \boldsymbol{Y})$ w.r.t. $\boldsymbol{\theta}$ using some iterations of a quasi-Newton procedure starting from $\boldsymbol{\theta}^{(l-1)}$. {Step 2 (Reallocation): } Set $h_{ti}^{(l)} := \begin{cases} 1 & \text{if } t = \max_{t'=1,\dots,T} \{t' \| L_{it'}(\boldsymbol{\theta}_t^{(l)} \| \boldsymbol{Y}) = \max_{t''=1,\dots,T} \{L_{it''} \boldsymbol{\theta}_t^{(l)} \| \boldsymbol{Y})\}\}, \\ 0 & \textit{otherwise}, \end{cases}$ $\forall t,i.$
Until	$-\ln L(\boldsymbol{\theta}^{(l)}, \boldsymbol{H}^{(l)} \| \boldsymbol{Y}) > -\ln L(\boldsymbol{\theta}^{(l-1)}, \boldsymbol{H}^{(l-1)} \| \boldsymbol{Y}) - \delta.$

or of CAIC (Consistent Akaike Information Criterion, see Bozdogan, 1987) of the type

$$\mathrm{CAIC} = -2\ln L(\hat{\boldsymbol{\theta}}, \boldsymbol{H}|\boldsymbol{Y}) + (1 + \ln \mathrm{NO})\mathrm{NEP}, \tag{12}$$

have been used. Here, NEP denotes the number of effective parameters, NO the number of observations, and $\hat{\boldsymbol{\theta}}$ the maximum likelihood estimate of $\boldsymbol{\theta}$, all with respect to some model under consideration. Even though the underlying regularity conditions for AIC and CAIC across different segmentation matrices $\boldsymbol{H}$ are not satisfied, these formulae have been proposed and are applied here in the same sense, see, e.g., Bozdogan (1987, 1993), and Wedel and DeSarbo (1993). Note that this external approach with fixed product coordinates does not suffer from the usual indeterminacies of MDS-type models. The number of effective ideal point parameters can be calculated using $\mathrm{NEP} = \mathrm{T}r(r + 1)/2$.

3.3. *Choice prediction and generation of positioning options*

Based on the estimated joint space parameters (shares of) choices for existing products can be predicted using Eqs. (2) and (4). Also, an extension of the just described situation to optimal positioning issues is possible: Let us assume that the product set S is enlarged by a new product 0 to $S_0 = S \cup \{0\}$. Using

$$p_{0|S_0}(\boldsymbol{x}_0) = \sum_{t=1}^{\mathrm{T}} \lambda_t p_{t0|S_0}(\boldsymbol{x}_0) = \sum_{t=1}^{\mathrm{T}} \lambda_t \int_{z \in R_{0|S_0}(\boldsymbol{x}_0)} f_t(z)\,\mathrm{d}z \tag{13}$$

with

$$R_{0|S_0}(\boldsymbol{x}_0) = \{z \in \mathbb{R}^r \,|\, (z - \boldsymbol{x}_0)'(z - \boldsymbol{x}_0) \leqslant (z - \boldsymbol{x}_k)'(z - \boldsymbol{x}_k)\ \forall k \in S_0\}, \tag{14}$$

a calculation of the share of choices for the new product 0 positioned at the point $\boldsymbol{x}_0 = (x_{01}, \ldots, x_{0r})'$ can be performed. As in other optimal positioning procedures it is assumed that the new product can be made similarly well-known and distributed in the market under consideration as the existing products. Optimal positioning options for the new product can be obtained through maximizing Eq. (13) using a standard hill-climbing algorithm of nonlinear programming. Again, fifty restarts are used to tackle the problem of local optima.

4. Monte Carlo experiment

Whenever a new approach is presented comparisons with existing counterparts can be used to get a feeling how the proposed methodology 'behaves' and to demonstrate whether and to which extent improvements can be reported. Although several contributions are known that deal with such comparisons in the here described area we just refer to Sudharshan et al. (1987) and Vriens et al. (1996) because the design of the following Monte Carlo experiment was mainly influenced by these papers.

In Sudharshan et al. (1987)'s comparison of four MDS-based (heuristic) techniques for optimal product positioning – a grid search algorithm, PROPOSAS (Albers and Brockhoff, 1979), GHS-IV (Gavish et al. (1983)) and the own PRODSRCH method – were judged with respect to their ability to find positioning options that maximize share of choices. Additionally, Vriens et al. (1996)'s Monte Carlo comparison of various metric conjoint segmentation methods is a well-known example from the CA-based perspective.

Adopting much from the design of the studies just mentioned, a total of 320 data sets was generated according to a full factorial design with the following five factors: number of (simulated) respondents (100 and 200), number of products (8 and 10), number of (equally sized) segments (2 and 4), number of attributes (2 and 3) and heterogenity within segments (0.5 and 1.5 variance). Note that heterogenity allows to take into account varying degrees of inconsistencies in the generated data, e.g. intransitivities within the paired comparison experiment. Each factor-level combination was replicated 10 times. Segment-specific ideal points were assumed to be independently and identically normally distributed across attributes.

For the generation of each data set the following steps were used: First, for a product market with specified number of products, number of segments and number of attributes the product point and (expected) segment-specific ideal point coordinates were randomly drawn from a prespecified range (the interval $[-3, 3]$ was selected). Second, for the specified number of respondents (randomly assigned to equally sized segments) paired comparison data and ten holdout choices were generated through sampling an ideal point from the corresponding segment-specific ideal point distribution for each paired comparison and each holdout choice and assuming that the product with the nearest product point in the pair resp. in the whole set of products is selected.

Each generated data set was analyzed using the new approach and a traditional counterpart. In the new approach, the segmentation scheme and the segment-specific model parameters were estimated simultaneously according to the description of Section 3. To prevent local optima, 50 random starts were used. Shares of choices for the existing products were estimated according to Eqs. (2) and (4). Finally, an optimal positioning was estimated using Eq. (13). In the traditional counterpart, the paired comparison data were clustered using a k-means procedure before segment-specific ideal points were estimated by the probabilistic ideal point model from Table 1. Again, 50 random starts were used. Then, shares of choices for the existing products were predicted according to the 'probabilistic' choice rule. The (constant) utility weighting parameter was estimated so that the root-mean-squared error between the predicted shares of choices $p_{j|S}$ ($j \in S = \{1, \ldots, m\}$) and the (true) shares of choices according to the holdout choices from the data set $p^*_{j|S}$ was minimized (* denotes true values in the Monte Carlo simulation.). Finally, an optimal positioning option was estimated using a standard nonlinear programming technique with fifty random starts for deriving optimal new product positionings.

The results from the two approaches were evaluated according to the following seven performance measures PM_1 to PM_7, where – before evaluation – segments in the derived solutions were permuted in such a way that the corresponding segment-specific values were well matched:

- $PM_1 = \sqrt{\sum_{t=1}^{T}\sum_{p=1}^{r}(\mu_{tp} - \mu^*_{tp})^2/Tr}$ (root-mean-squared error between the true and estimated expected segment-specific ideal point coordinates),
- $PM_2 = \sqrt{\sum_{t=1}^{T}\sum_{p=1}^{r}(\sigma_{tpp} - \sigma^*_{tpp})^2/Tr}$ (root-mean-squared error between the true and estimated variance parameters for describing the distribution of the segment-specific ideal points),
- $PM_3 = \sum_{t=1}^{T}\sum_{i=1}^{N} h_{ti}h^*_{ti}/N$ (percentage of correctly classified respondents, which is the share of respondents that are assigned to their true segments),
- $PM_4 = \sqrt{\sum_{j=1}^{m}(p_{j|S} - p^*_{j|S})^2/m}$ (root-mean-squared error between the true and estimated shares of choices),
- $PM_5 = \sqrt{\sum_{t=1}^{T}\sum_{j=1}^{m}(p_{tj|S} - p^*_{tj|S})^2/Tm}$ (root-mean-squared error between the true and estimated segment-specific shares of choices),
- $PM_6 = \sqrt{\sum_{p=1}^{r}(x_{0p} - x^*_{0p})^2/r}$ (root-mean-squared error between the 'true' and estimated coordinates for the optimal positioned new product), and
- $PM_7 = \sqrt{(p_{0|S_0} - p^*_{0|S_0})^2}$ (root-mean-squared error between the 'true' and estimated share of choices for the optimal positioned new product).

For PM_6 and PM_7 'true' (in quotation marks) means that these values were obtained by applying the new approach on the basis of the true model parameters mentioned for the description of the performance measures PM_1 to PM_3.

PM_1, PM_2, and PM_3 help to explain what could be called 'parameter recovery' of the ideal point model used, PM_4 and PM_5 are related to 'choice prediction' with respect to the existing products under competition, while PM_6 and PM_7 give hints concerning 'new product detection'. Of course, our new approach should show advantages with respect to new product detection and choice prediction while significant differences concerning parameter recovery were of minor interest.

As outcome of the Monte Carlo experiment, 640 observations for each of the seven performance measures (from 320 generated product-markets according to the 32 factor-level-combinations with ten replications, analyzed by the two approaches) were available in order to show differences across approaches and factors used for the generation of the product-markets. Table 5 shows mean

Table 5
Mean values for the performance measures under different approaches and outcomes of factors

	PM_1	PM_2	PM_3	PM_4	PM_5	PM_6	PM_7
Approach Trad.	0.2288	0.3202	0.9537	0.0382[a]	0.0705[a]	0.7244[a]	0.1260[a]
New	0.2236	0.3309	0.9558	0.0121	0.0238	0.1596	0.0219
Respond. 100	0.2574[a]	0.3953[a]	0.9542	0.0262	0.0490	0.4919	0.0766
200	0.1949	0.2558	0.9553	0.0241	0.0454	0.3921	0.0713
Products 8	0.2798[a]	0.3872[a]	0.9456[a]	0.0268[a]	0.0514[a]	0.5084[a]	0.0720
10	0.1725	0.2639	0.9638	0.0235	0.0430	0.3756	0.0758
Segments 2	0.1574[a]	0.2431[a]	0.9777[a]	0.0263	0.0400[a]	0.4954	0.0915[a]
4	0.2949	0.4080	0.9318	0.0240	0.0543	0.3887	0.0564
Attrib. 2	0.2068[a]	0.2747[a]	0.9445[a]	0.0262	0.0498[a]	0.4596	0.0754
3	0.2455	0.3764	0.9650	0.0241	0.0445	0.4245	0.0725
Heterog. Hom.	0.1942[a]	0.1753[a]	0.9639[a]	0.0232[a]	0.0440[a]	0.3696[a]	0.0591[a]
Het.	0.2582	0.4758	0.9456	0.0271	0.0503	0.5144	0.0888

[a]Indicates that the difference between the two means is significant at the 0.01 level.

values for the performance measures under different approaches and outcomes of factors while in Table 6 the mean values for the performance measures under different approaches by outcomes of factors are depicted.

Additionally, for the five Monte Carlo factors and the approaches as main effects as well as for those first-order interaction effects in which approaches constitute one of the interacting parts, F-test results with respect to all performance measures were checked in an ANOVA context and are – as they are in agreement with the information contained in Tables 5 and 6 – not reported.

Altogether, the message from the Monte Carlo experiment is pretty clear: With respect to choice prediction (PM_4, PM_5) and new product detection (PM_6, PM_7) the new approach performs significantly (at the 0.01 level) better than the traditional counterpart. For these performance measures also the first-order interaction effects reveal significant (although not all at the 0.01 level) differences and Table 6 shows that application of the new approach contributes more than the variation of the factors for generating the product-markets to overall improvements. With respect to parameter recovery (PM_1, PM_2, PM_3) the hypothesis that the two approaches perform equally well cannot be rejected. Also, all first-order interaction effects do not show significance. This supports our assumption that both approaches are equally well suited for the parameter recovery of the underlying ideal point model (i.e., we did not select a 'poor' traditional approach to gain advantages for our) and that a main reason for the better performance of the new approach is due to the fact that the assumptions for modeling consumer's choice decisions and for generating product positioning options are the same.

To conclude, the new approach outperforms the traditional counterpart with respect to the prediction of shares of choices for existing and new products as

Table 6
Mean values for the performance measures under different approaches by outcomes of factors

		Respond.		Products		Segments		Attrib.		Heterog.	
	Appr.	100	200	8	10	2	4	2	3	Hom.	Het.
PM_1	Trad.	0.2550^{b}	0.2025	0.2799^{b}	0.1776	0.1630^{b}	0.2945	0.2087	0.2488	0.1971^{b}	0.2604
	New	0.2598^{b}	0.1873	0.2798^{b}	0.1673	0.1519^{b}	0.2952	0.2050	0.2421	0.1912^{b}	0.2559
PM_2	Trad.	0.3806^{b}	0.2597	0.3779^{b}	0.2625	0.2440^{b}	0.3963	0.2649^{b}	0.3754	0.1781^{b}	0.4622
	New	0.4099^{b}	0.2519	0.3965^{b}	0.2653	0.2421^{b}	0.4198	0.2845^{b}	0.3774	0.1724^{b}	0.4894
PM_3	Trad.	0.9545	0.9529	0.9446	0.9628	0.9764^{b}	0.9309	0.9434^{b}	0.9639	0.9627	0.9447
	New	0.9539	0.9577	0.9467	0.9648	0.9789^{b}	0.9326	0.9456	0.9660	0.9650	0.9466
PM_4	Trad.	0.0383^{a}	0.0380^{a}	0.0402^{a}	0.0361^{a}	0.0409	0.0354^{a}	0.0414^{a,b}	0.0349^{a}	0.0345^{a,b}	0.0418^{a}
	New	0.0141^{b}	0.0102	0.0133^{b}	0.0110	0.0118	0.0125	0.0110^{b}	0.0133	0.0118	0.0125
PM_5	Trad.	0.0701^{a}	0.0710^{a}	0.0761^{a,b}	0.0650^{a}	0.0619^{a,b}	0.0792^{a}	0.0762^{a,b}	0.0649^{a}	0.0644^{a,b}	0.0767^{a}
	New	0.0278^{b}	0.0198	0.0266^{b}	0.0210	0.0181^{b}	0.0295	0.0234	0.0242	0.0237	0.0239
PM_6	Trad.	0.8152^{a}	0.6336^{a}	0.8090^{a}	0.6398^{a}	0.8142^{a}	0.6346^{a}	0.7973^{a}	0.6515^{a}	0.6186^{a}	0.8302^{a}
	New	0.1687	0.1506	0.2079	0.1114	0.1765	0.1427	0.1219	0.1974	0.1207	0.1986
PM_7	Trad.	0.1271^{a}	0.1248^{a}	0.1178^{a}	0.1342^{a}	0.1593^{a,b}	0.0927^{a}	0.1363^{a}	0.1157^{a}	0.0972	0.1547^{a}
	New	0.0260	0.0178	0.0263^{b}	0.0175	0.0237	0.0202	0.0145^{b}	0.0294	0.0209	0.0229

^{a,b} Indicates that the difference between the two means with respect to the approaches (outcomes of factors) is significant at the 0.01 level.

well as with respect to the detection of optimal positioning options for new products. We leave an interpretation of further results contained in Tables 5 and 6 to the reader. Instead, in the following section, we show how the new approach works based on a subsample from a real-world situation which we are allowed to use for demonstration purposes.

5. Application

Paired comparison data from a sample of 56 respondents, who were asked for preference judgments related to 8 coffee brands denoted by 'brand A' to 'brand H', are used as well as individual perceptual judgments for each brand with respect to the properties 'special flavor', 'high quality', 'good taste', 'unblended', 'no bitter constituent parts', 'mild', 'decoffeinated', 'bitter', and 'a blend of different types'. Two-dimensional and three-dimensional representations of the brands in a perceptual space via principal component analysis with varimax rotation constitute starting points for the following external analyses. The first two factors explained 78.52%, the first three factors 90.01% of the variance of the (aggregated) perceptual data. The new approach described in Section 3 was used to determine segmentation schemes for the respondents, (expected) segment-specific ideal points and covariance matrices for $T = 1, 2, \ldots, 10$ segments. Table 7 shows the results in terms of $-\ln L$, AIC, and CAIC. We see that the CAIC values indicate that the two-dimensional representation and the 5-segment solution should be preferred.

In Fig. 1, the 5-segment solution of our approach is visualized in a joint space in which property vectors, preference subsets of the brands, and 75%-confidence regions for the expected ideal points help to explain the underlying situation. The confidence regions refer to the maximum likelihood estimates of the segment-specific ideal point distribution parameters. Some obvious results are: The first factor combines properties as 'mild', 'no bitter constituent parts', and 'decoffeinated' in its positive direction at the right hand side while 'bitter' explains its negative direction. The second factor describes a quality dimension with properties as 'high quality', 'good taste', 'special flavor' and to some extent 'unblended' in its positive direction and 'a blend of different types' in the opposite direction.

'Brand D', 'brand G', and 'brand B' are rated highest on the positive direction of the first factor, 'brand H' and 'brand G' are to a much higher extent than the other brands characterized by the negative direction of the second factor, and so on. From the expected ideal point coordinates and their 75%-confidence regions we get an impression concerning the segment-specific preferences of the respondents, e.g., 'segm.1' likes brands with higher ratings on the positive directions of the first and second factor while 'segm.2' accepts brands with

Table 7
Summary of selected analyses using the new approach

T	r	NEP	$-\ln L$	AIC	CAIC
1	2	4	261.4	530.7	550.7
2	2	8	221.4	458.8	498.8
3	2	12	183.7	391.3	451.2
4	2	16	172.3	376.6	456.5
5	2	20	154.3	348.6	<u>448.4</u>
6	2	24	144.1	336.2	456.0
7	2	28	131.0	318.0	457.8
8	2	32	124.4	312.8	472.6
9	2	36	121.5	315.1	494.8
10	2	40	115.6	<u>311.1</u>	510.8
T	r	NEP	$-\ln L$	AIC	CAIC
1	3	6	252.1	516.1	546.1
2	3	12	202.7	429.4	489.3
3	3	18	176.6	389.1	479.0
4	3	24	167.7	383.3	503.1
5	3	30	150.7	361.4	511.2
6	3	36	140.1	352.2	531.9
7	3	42	128.8	341.6	551.3
8	3	48	123.9	343.9	583.5
9	3	54	118.6	345.3	614.8
10	3	60	<u>112.7</u>	345.3	644.8

$N = 56$; NEP $= 2Tr$; underline denotes best performance.

higher ratings on their opposite directions. The preference subsets of the brands support these findings.

Now, the formulation of our approach described above allows the prediction of shares of choices for the existing brands according to Eqs. (2) and (4). The results are shown in Table 8 as segment-specific and total values. We see – what we already know from Fig. 1 – that 'segm.2' is 'nearest' to 'brand H' in terms of segment-specific shares of choices but the new approach is flexible enough to incorporate the fact that, e.g., 'brand A', 'brand B', and 'brand G' can attract 7%, 8%, and 8% of the choices of 'segm.2'.

An answer to the interesting question, where a new product could be (optimally) positioned, is given by Fig. 2 and Table 9. The lines of the contour plot indicate the shares of choices reachable when a respective positioning option for a new product is selected. A comparison of Tables 8 and 9 shows that 'segm.1' and 'segm.3' or – in other words – mainly 'brand E', 'brand B', and 'brand D' contribute to the share of choices for the new product when the optimal positioning option is selected.

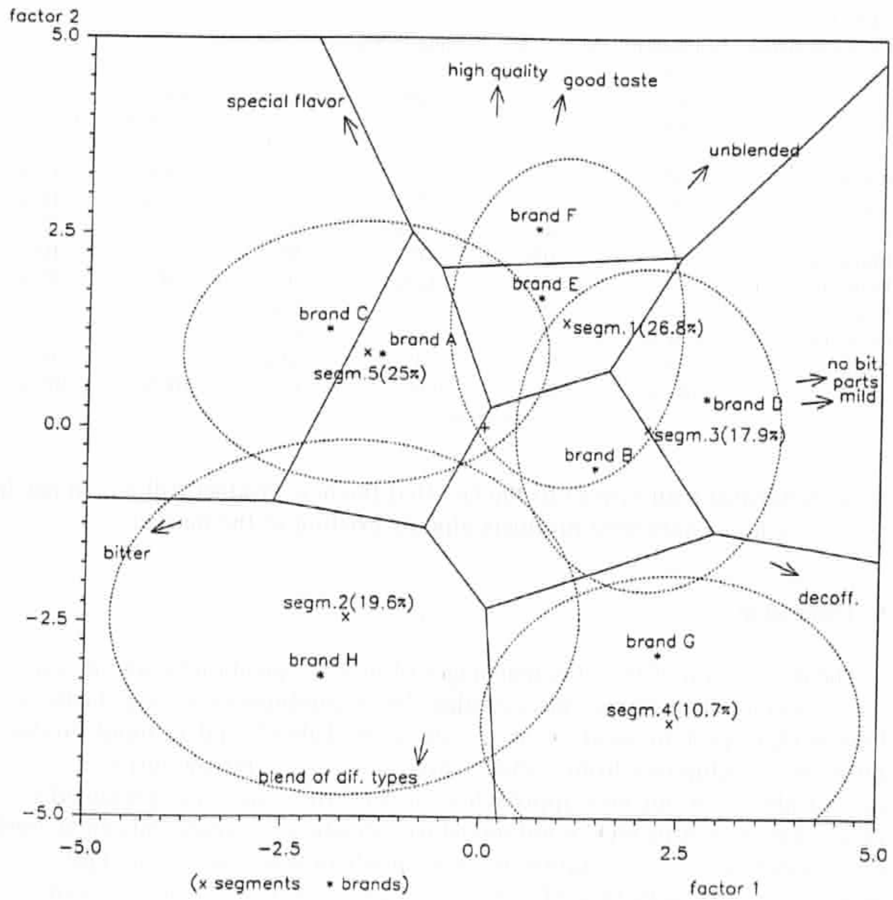

Fig. 1. Joint space solution with property vectors, brand points and expected segment-specific ideal points with 75%-confidence regions as well as preference subsets of the brands.

Predictions of this kind assume – as already mentioned in Section 3 – that the new product is similarly well-known and distributed in the market under consideration as the established brands, a situation which usually highly depends on the activities (spendings, time) of the introducing enterprise. Thus, the maximum value of 12% for the share of choices of the new product in Table 9 can be regarded as an upper bound obtainable under favorable conditions.

To conclude, the results of the empirical example demonstrate that the new approach allows the prediction of shares of choices for a new product for various positionings. The share of choices for the optimal positioning option as well as for other options can be regarded as an upper bound. Adjustments have

Table 8
Predicted shares of choices for the brands according to Fig. 1

	Segm.1 (26.8%)	Segm.2 (19.6%)	Segm.3 (17.9%)	Segm.4 (10.7%)	Segm.5 (25.0%)	Total
Brand A	5%	7%	1%	0%	38%	12%
Brand B	19%	8%	37%	4%	4%	15%
Brand C	0%	4%	0%	0%	43%	12%
Brand D	10%	0%	41%	2%	0%	10%
Brand E	41%	0%	11%	0%	10%	15%
Brand F	25%	0%	2%	0%	4%	8%
Brand G	0%	8%	8%	90%	0%	13%
Brand H	0%	73%	0%	4%	1%	15%
Total	100%	100%	100%	100%	100%	100%

to be performed with respect to the fact that the new product still has to reach the strength of established products already existing in the market.

6. Discussion

The description of the principal stages of product positioning and design in the beginning of this paper has revealed that a combination of, e.g., methodology with respect to ideal point models from Table 1 and optimal product positioning techniques from Table 3 should be helpful for the introduction of new products. In our new approach a model formulation was presented that relates a probabilistic ideal point model with positioning issues and can be used for as well parameter estimation as choice prediction and detection of positionings for new products. In a Monte Carlo experiment it was demonstrated that our approach competes favorably with a counterpart in which methodology from Tables 1 and 3 is used in a traditional way. The traditional counterpart was outperformed with respect to the prediction of shares of choices for existing and new products as well as with respect to the detection of optimal positioning options for new products. Based on a subsample from a real-world example we tried to provide a feeling 'how the new approach behaves'. Of course, against the background of literature as listed in Tables 1–3 possibilities for further extensions can be mentioned, e.g.:

- The presented formulation can be modified to handle paired comparison data from conjoint experiments where product coordinates (attribute-level combinations for the conjoint stimuli) vary systematically.
- The current objective function can be replaced if, e.g., sales volume or profit is of priority importance.

Activities concerning research in the just mentioned directions are on the way.

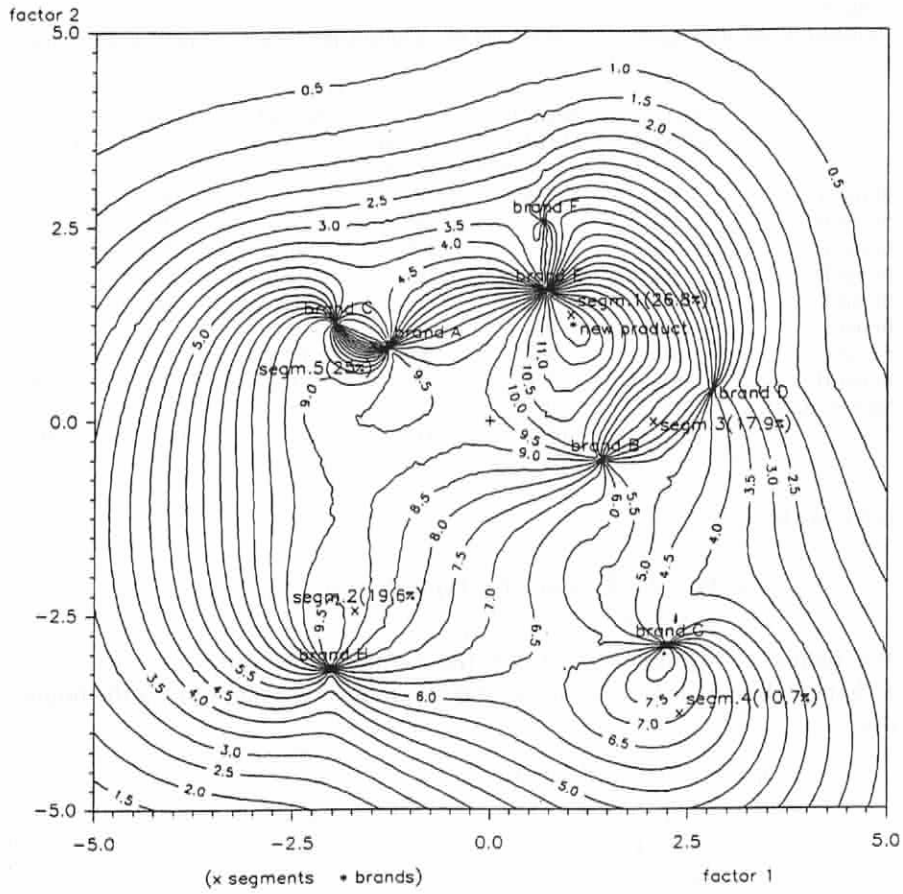

Fig. 2. Contour plot for positioning options and optimally positioned new product.

Appendix

Here, the hypercube approximation for Eq. (2) is described. Two cases are distinguished: First, it is assumed that the expected ideal point for segment t is located in the origin and that the segment-specific covariance matrix is diagonal. Then, it is shown how the general case can be transformed to the situation explained before.

Case a: ($\boldsymbol{\mu}_t = \boldsymbol{0}$, $\boldsymbol{\Sigma}_t = \text{diag}(\sigma_{t11}, \ldots, \sigma_{trr})$). Note that

$$\int_{a_1}^{b_1} \cdots \int_{a_r}^{b_r} f_t(z_1, \ldots, z_r)\, dz_r \cdots dz_1 = \prod_{p=1}^{r} (F_{tp}(b_p) - F_{tp}(a_p))$$

Table 9
Predicted shares of choices for the brands and the optimally positioned new product according to Fig. 2

	Segm.1 (26.8%)	Segm.2 (19.6%)	Segm.3 (17.9%)	Segm.4 (10.7%)	Segm.5 (25.0%)	Total
Brand A	5%	7%	1%	0%	38%	12%
Brand B	15%	8%	34%	4%	3%	13%
Brand C	0%	4%	0%	0%	43%	12%
Brand D	7%	0%	38%	2%	0%	9%
Brand E	17%	0%	2%	0%	6%	6%
Brand F	25%	0%	2%	0%	4%	8%
Brand G	0%	8%	8%	90%	0%	13%
Brand H	0%	73%	0%	4%	1%	15%
New product	31%	0%	15%	0%	5%	12%
Total	100%	100%	100%	100%	100%	100%

holds with

$$F_{tp}(z_p) = \Phi\left(\frac{z_p}{\sqrt{\sigma_{tpp}}}\right) \quad \text{and} \quad F_{tp}^{-1}(x) = \Phi^{-1}(x)\sqrt{\sigma_{tpp}}.$$

We define an 'iso-mass' grid for the r-dimensional attribute space with L^r hypercubes (with $L \in \mathbb{N}$ as a parameter to adjust granularity) via the boundaries

$$\zeta_{tl_p} = \begin{cases} -\infty & \text{if } l_p = 0, \\ F_{tp}^{-1}(l_p/L) & \text{if } l_p = 1, \ldots, L-1, \quad \forall t, p, l_p = 0, \ldots, L \\ \infty & \text{if } l_p = L, \end{cases}$$

and get the 'iso-mass' condition

$$\int_{\zeta_{t(l_1-1)}}^{\zeta_{tl_1}} \cdots \int_{\zeta_{t(l_r-1)}}^{\zeta_{tl_r}} f_t(z_1, \ldots, z_r)\, \mathrm{d}z_r \cdots \mathrm{d}z_1 = \frac{1}{L^r}$$

($L = 50$ was chosen throughout all analyses in this paper.).

Now, using the (probability mass) centroids of the hypercubes

$$\xi_{l_1 \ldots l_r} = \left(F_{t1}^{-1}\left(\frac{2l_1 - 1}{2L}\right), \ldots, F_{tr}^{-1}\left(\frac{2l_r - 1}{2L}\right)\right)'$$

as an (approximative) indicator whether all points of the hypercube are assumed to belong to $R_{j|S}$ or not, we can estimate $p_{tj|S}$ from Eq. (2) as the fraction of hypercubes for which $\xi_{l_1 \ldots l_r} \in R_{j|S}$ holds.

Case b: ($\boldsymbol{\mu}_t \in \mathbb{R}^r$, $\boldsymbol{\Sigma}_t$ positive definite). We use eigenvalue calculations for estimating a rotation matrix $\boldsymbol{C}_t$ ($\boldsymbol{C}_t'\boldsymbol{C}_t = \text{diag}(1,\dots,1)$) for which

$$\boldsymbol{\Sigma}_t = \boldsymbol{C}_t \text{diag}(v_{t1},\dots,v_{tr})\boldsymbol{C}_t'$$

holds. The transformation $z \to \tilde{z} = \boldsymbol{C}_t'(z - \boldsymbol{\mu}_t)$ leads to

$$\tilde{\boldsymbol{x}}_j = \boldsymbol{C}_t'(\boldsymbol{x}_j - \boldsymbol{\mu}_t),$$
$$\tilde{\boldsymbol{\mu}}_t = \boldsymbol{0},$$
$$\tilde{\boldsymbol{\Sigma}}_t = \text{diag}(v_{t1},\dots,v_{tr}) \text{ and}$$
$$\tilde{R}_{j|S} \text{ (analogous to } R_{j|S})$$

that describe a situation for which case a can be applied.

References

Akaike, H., 1977. On entropy maximization principle. In: Krishnaiah, P.R. (Ed.), Applications of Statistics. North-Holland, Amsterdam, pp. 27–41.

Akao, Y., 1990. QFD, Integrating Customer Requirements into Product Design. Productivity Press, Cambridge, MA.

Albers, S., 1977. A mixed integer nonlinear programming procedure for simultaneously locating multiple products in an attribute space. Operations Research Verfahren 26, 899–909.

Albers, S., 1989. Gewinnorientierte Neuproduktpositionierung in einem Eigenschaftsraum. Zeitschrift für betriebswirtschaftliche Forschung 41, 186–209.

Albers, S., Brockhoff, K., 1979. A comparison of two approaches to the optimal positioning of a new product in an attribute space. Zeitschrift für Operations Research 23, 127–142.

Bachem, A., Simon, H., 1981. A product positioning model with costs and prices. European Journal of Operational Research 7, 362–370.

Baier, D., Gaul, W., 1996. Analyzing paired comparisons data using probabilistic ideal point and vector models. In: Bock, H.H., Polasek, P. (Eds.), Data Analysis and Information Systems. Springer, Berlin, 163–174.

Balakrishnan, P.V., Jacob, V.S., 1996. Genetic algorithms for product design. Management Science 42, 1105–1117.

Böckenholt, I., 1989. Mehrdimensionale Skalierung qualitativer Daten. Lang, Frankfurt.

Böckenholt, I., Böckenholt, U., 1991. Constrained latent class analysis: Simultaneous classification and scaling of discrete choice data. Psychometrika 56, 699–716.

Böckenholt, I., Gaul, W., 1986. Analysis of choice behaviour via probabilistic ideal point and vector models. Applied Stochastic Models and Data Analysis 2, 209–226.

Böckenholt I., Gaul, W., 1988. Probabilistic multidimensional scaling of paired comparisons data. In: Bock, H.H. (Ed.), Classification and Related Methods of Data Analysis. North-Holland, Amsterdam, pp. 405–412.

Böckenholt, I., Gaul, W., 1989. Generalized latent class analysis: A new methodology for market structure analysis. In: Opitz, O. (Ed.), Conceptual and Numerical Analysis of Data. Springer, Berlin, pp. 367–376.

Bozdogan, H., 1987. Model selection and akaike's information criterion: The general theory and its analytical extensions. Psychometrika 52, 345–370.

Bozdogan, H., 1993. Choosing the number of component clusters in the mixture-model using a new informational complexity criterion of the inverse-Fisher information matrix. In: Opitz, O., Lausen, B., Klar, R. (Eds.), Information and Classification. Springer, Berlin, pp. 40–54.
Brunswick, E., 1952. The Conceptual Framework of Psychology. University of Chicago Press, Chicago, IL.
Bryant, P.G., Williamson, J.A., 1978. The asymptotic behavior of classification maximum likelihood estimates. Biometrika 65, 273–281.
Bryant P.G., Williamson, J.A., 1986. Maximum likelihood and classification: A comparison of three approaches. In: Gaul, W., Schader, M. (Eds.), Classification as a Tool of Research. North-Holland, Amsterdam, pp. 35–45.
Carmone, F.J., Green, P.E., Jain, A.K., 1978. Robustness of conjoint analysis: Some Monte Carlo results. Journal of Marketing Research 15, 300–303.
Carroll, J.D., 1997. Psychometric methods in marketing research: Part II, multidimensional scaling. Journal of Marketing Research 34, 193–204.
Choi, S., DeSarbo, W.S., 1993. Game theoretic derivations of competitive strategies in conjoint analysis. Marketing Letters 4, 337–348.
Choi, S., DeSarbo, W.S., 1994. A conjoint-based product designing procedure incorporating price competition. Journal of Product Innovation Management 11, 451–459.
Choi, S., DeSarbo, W.S., Harker, P.T., 1990. Product positioning under price competition. Management Science 36, 175–199.
Choi, S., DeSarbo, W.S., Harker, P.T., 1992. A numerical approach to deriving long-run equilibrium solutions: A research note. Management Science 38, 75–86.
Cooper, L.G., Nakanishi, M., 1983. Two logit models for external analysis of preferences. Psychometrika 48, 607–620.
DeSarbo, W.S., DeSoete, G., Eliashberg, J., 1987. A new stochastic multidimensional unfolding model for the investigation of paired comparison consumer preference/choice data. Journal of Economic Psychology 8, 357–384.
DeSarbo, W.S., DeSoete, G., Jedidi, K., 1987. Probabilistic multidimensional scaling models for analyzing consumer choice behavior. Communication & Cognition 20, 93–116.
DeSarbo, W.S., Hoffman, D.L., 1987. Constructing joint spaces from binary choice data: A multidimensional unfolding threshold model for marketing research. Journal of Marketing Research 24, 40–54.
DeSarbo, W.S., Ramaswamy, V., Cohen, S.H., 1995. Market segmentation with choice-based conjoint analysis. Marketing Letters 6, 137–147.
DeSarbo, W.S., Rao, V.R., 1986. A constrained unfolding methodology for product positioning. Marketing Science 5, 1–19.
DeSarbo, W.S., Wedel, M., Vriens, M., Ramaswamy, V., 1992. Latent class metric conjoint analysis. Marketing Letters 3, 273–288.
DeSoete, G., 1990. A latent class approach to modeling pairwise preferential choice data. In: Schader, M., Gaul, W. (Eds.), Knowledge, Data, and Computer-Assisted Decisions. Springer, Berlin, pp. 240–249.
DeSoete, G., Carroll, J.D., DeSarbo, W.S., 1986. The wandering ideal point model: A probabilistic multidimensional unfolding model for paired comparisons data. Journal of Mathematical Psychology 30, 28–41.
Dillon, W.R., Kumar, A., de Borrero, M., 1993. Capturing individual differences in paired comparisons: An extended BTL model incorporating descriptor variables. Journal of Marketing Research 30, 42–51.
Dobson, G., Kalish, S., 1988. Positioning and pricing a product Line. Marketing Science 7, 107–125.
Dobson, G., Kalish, S., 1993. Heuristics for pricing and positioning a product-line using conjoint and cost data. Management Science 39, 160–175.
Gaul, W., Aust, E., Baier, D., 1995. Gewinnorientierte Produktliniengestaltung unter Berücksichtigung des Kundennutzens. Zeitschrift für Betriebswirtschaftslehre 65, 835–854.

Gavish, B., Horsky, D., Srikanth, K., 1983. An approach to the optimal positioning of a new product. Management Science 29, 1277–1297.

Green, P.E., 1984. Hybrid models for conjoint analysis: An expository review. Journal of Marketing Research 21, 155–159.

Green, P.E., Carroll, J.D., Goldberg, S.M., 1981. A general approach to product design via conjoint analysis. Journal of Marketing 45, 17–37.

Green, P.E., Krieger, A.M., 1985. Models and heuristics for product line selection. Marketing Science 4, 1–19.

Green, P.E., Krieger, A.M., 1989. Recent contributions to optimal product positioning and buyer segmentation. European Journal of Operational Research 41, 127–141.

Green, P.E., Krieger, A.M., 1992. An application of a product positioning model to pharmaceutical products. Marketing Science 11, 117–132.

Gupta, S.S., 1963. Probability integrals of multivariate normal and multivariate t. Annals of Mathematical Statistics 34, 792–828.

Gustafsson, A., 1996. Customer Focused Product Development by Conjoint Analysis and QFD. Linkoeping University Press, Linkoeping, Sweden.

Hauser, J.R., Clausing, D., 1988. The house of quality. Harvard Business Review 66, 63–73.

Horsky, D., Nelson, P., 1992. New brand positioning and pricing in an oligopolistic market. Marketing Science 11, 133–153.

Huber, J., Holbrook, M.B., 1979. Using attribute ratings for product positioning: Some distinctions among compositional approaches. Journal of Marketing Research 16, 507–516.

Johnson R.M., 1987. Adaptive conjoint analysis. In: 1987 Sawtooth Conference Proceedings. Sawtooth Software Inc., Evanston, IL, pp. 253–265.

Kamakura, W.A., 1988. A least squares procedure for benefit segmentation with conjoint experiments. Journal of Marketing Research 25, 157–167.

Kamakura, W.A., Srivastava, R.K., 1986. An ideal-point probabilistic choice model for heterogeneous preferences. Marketing Science 5, 199–218.

Kamakura, W.A., Wedel, M., Agrawal, J., 1994. Concomitant variable latent class models for conjoint analysis. International Journal for Research in Marketing 11, 451–464.

Kaul, A., Rao, V.R., 1995. Research for product positioning and design decisions: An integrative review. International Journal of Research in Marketing 12, 293–320.

Kohli, R., Krishnamurti, R., 1987. A heuristic approach to product design. Management Science 33, 1523–1533.

Kohli, R., Sukumar, R., 1990. Heuristics for product-line design using conjoint analysis. Management Science 36, 1464–1478.

Krishnaiah, P.R., 1980. Computations of some multivariate distributions. In: Krishnaiah, P.R. (Ed.), Handbook of Statistics, vol. 1. North-Holland, Amsterdam, pp. 745–971.

Kruskal, J.B., 1965. Analysis of factorial experiments by estimating monotone transformations of the data. Journal of the Royal Statistical Society 27, 251–263.

Louviere, J.J., Woodworth, G.G., 1983. Design and analysis of simulated choice or allocation experiments: An approach based on aggregate data. Journal of Marketing Research 20, 350–367.

McBride, R.D., Zufryden, F.S., 1988. An integer programming approach to optimal product-line selection. Marketing Science 7, 126–140.

Narasimhan, C., Sen, S., 1989. Linking engineering attributes to perceptual characteristics. Working paper, Yale School of Management, New Haven, NY.

Sawtooth Software Inc., 1993. CBC System Version 1.0, Sawtooth Software Inc., Evanston, IL.

Sawtooth Software Inc., 1994. ACA System Version 4.0. Sawtooth Software Inc., Evanston, IL.

Schmalensee, R., Thisse, J.F., 1988. Perceptual maps and the optimal location of new products: An integrative essay. International Journal of Research in Marketing 5, 225–249.

Shocker, A.D., Srinivasan, V., 1974. A consumer-based methodology for the identification of new product ideas. Management Science 20, 921–937.

Shocker, A.D., Srinivasan, V., 1979. Multiattribute approaches for product concept evaluation and generation: A critical review. Journal of Marketing Research 16, 159–180.
Srinivasan, V., Shocker, A.D., 1973. Linear programming techniques for multidimensional analysis of preferences. Psychometrika 38, 337–369.
Sudharshan, D., Kumar, K.R., Gruca, T., 1995. NICHER: An approach to identifying defensible product positions. European Journal of Operational Research 45, 292–389.
Sudharshan, D., May, J.H., Gruca, T., 1988. DIFFSTRAT: An analytical procedure for generating optimal new product concepts for a differentiated-type strategy. European Journal of Operational Research 36, 50–65.
Sudharshan, D., May, J.H., Shocker, A.D., 1987. A simulation comparison of methods for new product location. Marketing Science 6, 182–201.
Urban, G.L., Hauser, J.R., 1993. Design and Marketing of New Products. 2nd ed. Prentice Hall, Englewood Cliffs, NJ.
Vriens, M., Wedel, M., Wilms, T., 1996. Metric conjoint segmentation methods: A Monte Carlo comparison. Journal of Marketing Research 33, 73–85.
Wedel, M., DeSarbo, W.S., 1993. A latent class binomial logit methodology for the analysis of paired comparison choice data. Decision Sciences 24, 1157–1170.
Wedel, M., DeSarbo, W.S., 1996. An exponential family mixture MDS methodology for simultaneous segmentation and product positioning. Journal of Business & Economic Statistics 14, 447–459.
Wedel, M., Kistemaker, C., 1989. Consumer benefit segmentation using clusterwise linear regression. International Journal of Research in Marketing 6, 45–49.
Wedel, M., Steenkamp, J.B., 1989. Fuzzy clusterwise regression approach to benefit segmentation. International Journal of Research in Marketing 4, 241–258.
Wittink, D.R., Vriens, M., Burhenne, W., 1994. Commercial use of conjoint analysis in Europe: Results and critical reflections. International Journal of Research in Marketing 11, 41–52.
Zufryden, F.S., 1979. ZIPMAP – a zero-one integer programming model for market segmentation and product positioning. Journal of Operational Research Society 30, 63–70.