Product Bundling as a Marketing Application

Bernd Stauß and Wolfgang Gaul

Institut für Entscheidungstheorie und Unternehmensforschung
Universität Karlsruhe, P.O. Box 6980, 76128 Karlsruhe, Germany
{bernd.stauss, wolfgang.gaul}@wiwi.uni-karlsruhe.de

Abstract. Product bundling describes an interdisciplinary problem of great importance. It can be used to tailor offers to the demand of consumer segments (marketing), it helps to tackle variety reduction management issues (production), it is based on consumer preferences (data analysis), and it needs combinatorial optimization as solution tool (operations research).
In this paper a new profit-maximizing mixed integer product bundling formulation is presented that works well for modest problem instances. Additionally, a heuristic approach is derived that copes with the situation in a Greedy-like manner for larger problem instances by providing a sequence of monotone increasing lower bounds for the objective function of our product bundling methodology.

1 Introduction

The joint offer of two or more different products or services that are sold at a unique price is referred to as bundling. It was first discussed in the early 1960 in a primary legal context with a main focus on monopolists that attempted to expand their monopoly power into other competitive markets by the use of bundling ([13]). By the time two basically different research streams emerged: A more behavioristic orientated group of authors explored perceptional aspects of bundling and associated consequences on consumer preferences ([17], [9], [3]), another direction of research focused essentially on the determination of optimal bundling and pricing strategies. Like the approach of [1], researchers as [10] or [11] provided a graphical framework that allowed for analyzing different bundling strategies where mostly additive structures of product preferences were assumed (e.g., [1], [12], [11]). Motivations for bundling include exploitation of demand complementary, the already mentioned competitive effects, and the consideration of price discrimination. As a major advantage in terms of product variety many authors discussed the possibility of smoothing out consumer preferences as effect inherent in bundling . Some authors like [2] extended their work to situations where competition among firms is prevalent. However, little work has been done up to now concerning the development of usable decision models and appropriate algorithms for generating optimal bundles and prices, respectively. Main contributions in this area were given by the work of [6] and [16].

Since products can be often viewed as bundles of product characteristics ([15]), product bundling received growing attention in new product development due to its ability to tackle the problem of increasing product variety

([4]). As in many cases firms produce a basic version of a certain product which is offered along with several optional features, product variety is mainly generated by mixing and matching those options ([14]). This is fairly common practice, e.g., in the automobile or computer industry.

The remainder of this paper is structured as follows. Section 2 is intended to give an introduction into the fundamentals of a mathematical formulation for generating optimal bundles and prices, respectively. We discuss characteristics as described in the well-known Hanson, Martin paper ([6]) and explain possible problems that may arise when heuristic approaches are formulated. In section 3 a new model is proposed that enables a proper specification of a new heuristic approach and an example is presented to demonstrate its usefulness. Section 4 concludes the paper.

2 Fundamentals of a Mathematical Formulation of Product Bundling

2.1 Assumptions and Optimization Approach

Some preliminary assumptions and notations are needed to establish an underlying framework for product bundling. Starting point is the set of products J whose elements are also called components of the bundles to be created, $\mathcal{B}_l \subseteq J$ is the set of components contained in bundle l, L is the set of all possible bundles, and I is the set of segments of potential buyers (with $i = 1, \ldots, |I|$, $j = 1, \ldots, |J|$, and $l = 1, \ldots, |L| = 2^{|J|} - 1$). Since each product by itself constitutes a bundle, we assume that the first $|J|$ bundles in L correspond to the respective products. The cost c_l of bundle l is the sum of costs of the products contained in the bundle, i.e. $c_l = \sum_{j \in \mathcal{B}_l} c_j$. Bundle l can be offered to segment i at a price p_{il}. Each segment i is characterized by its size N_i and utilities or reservation prices r_{il} for every possible bundle l, i.e. utility is measured in monetary units via reservation prices.[1] Given prices, reservation prices, and the composition of the bundles, every consumer of segment i wants to maximize her/his surplus. Consumer surplus can be described as the difference between reservation price and actual price of the bundle l: $r_{il} - p_{il}$. We assume that consumers cannot get any surplus if they don't purchase anything and that the benefit obtained from multiple components is zero, because to avoid arbitrage we suppose that a resale of components is not possible. On the other side, we assume that there is free disposal of unwanted components.

Furthermore, consumer choice is denoted by binary variables $\theta_{il} \in \{0, 1\}$ with

$$\theta_{il} = \begin{cases} 1, & \text{if segment } i \text{ buys bundle } l \\ 0, & \text{otherwise.} \end{cases}$$

[1] A reservation price can be interpreted as the maximal price a consumer is willing to pay for a bundle.

Additionally, we define indicator variables $y_{jl} \in \{0,1\}$ where

$$y_{jl} = \begin{cases} 1, & \text{if } j \in \mathcal{B}_l \\ 0, & \text{otherwise} \end{cases}$$

which we will use in our new approach.

Explicit price discrimination is excluded so that all customers face identical prices for the same bundle (i.e. $p_{il} = p_l, i \in I, l \in L$). Despite the assumptions mentioned up to now we allow for combinations of bundles offered without any additional assembly costs. Without doubt the latter isn't consistent with the general assumption within select-one-out-of-many modeling of consumer choice behavior in traditional marketing models and constitutes a main characteristic of bundling. This fact should therefore receive particular attention in the sequel.

Definition 1. We call a price schedule subadditive if the following conditions hold:

$$p_l \leq \sum_{k \in K} p_k \quad l \in L, K \in \{\tilde{K} | \cup_{\tilde{k} \in \tilde{K}} \mathcal{B}_{\tilde{k}} = \mathcal{B}_l\}.$$

Under the above assumptions it can be shown that if a profit maximizing firm knows all relevant reservation prices of their potential customers, then there exists a subadditive price schedule which is optimal. As an immediate consequence one may conjecture that each consumer will purchase at most one of the bundles. A major drawback, however, is the fact that all possible bundles have to be considered explicitly.

Due to page restrictions we use the following MHM (Modified Hanson, Martin [6]) formulation:

MHM:

$$\max \sum_{i \in I} \sum_{l \in L} N_i (p_{il} - c_l \theta_{il}) \tag{1}$$

$$\sum_{\tilde{l} \in L} (r_{i\tilde{l}} \theta_{i\tilde{l}} - p_{i\tilde{l}}) \geq \max\{0, r_{il} - p_l\} \quad i \in I, l \in L \tag{2}$$

$$p_l \leq \sum_{k \in K} p_k \quad l \in L, K \in \{\tilde{K} | \cup_{\tilde{k} \in \tilde{K}} \mathcal{B}_{\tilde{k}} = \mathcal{B}_l\} \tag{3}$$

$$p_{il} \geq p_l - M(1 - \theta_{il}) \quad i \in I, l \in L \tag{4}$$

$$p_{il} \leq p_l \quad i \in I, l \in L \tag{5}$$

$$\sum_{l \in L} \theta_{il} \leq 1 \quad i \in I \tag{6}$$

$$p_l, p_{il} \geq 0 \quad i \in I, l \in L \tag{7}$$

$$\theta_{il} \in \{0,1\} \quad i \in I, l \in L \tag{8}$$

where M is a sufficiently large constant. The constraints allow for the following interpretation: The inequalities in (3) enforce a subadditive price schedule (see also definition 1) and consumer choice behavior resembles a first choice situation as given by constraint (2), i.e. each consumer will buy that bundle which provides the maximal surplus. Although the model allows for segment specific prices p_{il} each segment that actually buys faces identical prices for identical bundles (because if a bundle l is selected ($\theta_{il} = 1$), (4) and (5) enforce $p_{il} = p_l$, otherwise (for $\theta_{il} = 0$) conditions (2) along with the nonnegativity of p_{il} ensure $p_{il} = 0$). Constraints (6), (7), and (8) are self-explaining. As a solution of MHM we get an optimal set of bundles $L_{opt} = \{l \in L | \sum_{i \in I} \theta_{il} > 0\}$ that should be offered together with an optimal price schedule. It is worth mentioning that $|L_{opt}| \leq |I|$ holds as each segment chooses at most one of the bundles.

2.2 Price Subadditivity and Further Implications of MHM

A crucial part of MHM is constituted by constraint (3) that describes and guarantees for the special structure of the solution. However, Gaul, Stauß [5] show that it suffices to examine price subadditivity in terms of restrictions for pairs of bundles only instead of those given by (3).

Up to now no assumptions about the functional form of consumer preferences were made. One can tackle this problem by supposing that reservation prices for all bundles are known. But the availability of such information seems to be somewhat unrealistic given the combinatorial variety induced even by a small number of components. Thus, as it is frequently done in the literature, we suppose that reservation prices are additive, i.e. $r_{il} = \sum_{j \in B_l} r_{ij}$. Furthermore, this assumption enables the use of decompositional methods like conjoint analysis based on linear additive model specifications that provide estimators for reservation prices (see ,e.g., [8], [7] for a discussion of the methodology in the context of product bundling).

An essential drawback of MHM is given by the fact, that for optimization all possible bundles have to be listed explicitly. This is due to the necessity of checking price subadditivity in order to guarantee the appealing structure of the solution.

In the next section a heuristic approach is formulated in which constraints (2) and (3) of MHM are replaced by restrictions that are easier to handle with respect to an incomplete enumeration of bundles.

3 A New Model

3.1 Problem Reformulation

In a current paper of Gaul, Stauß [5] a new formulation of the bundling problem is proposed. The underlying assumptions of the new model are similar to those that were given in section 2 with a slight but essential modification: price subadditivity has not to be postulated explicitly anymore so that a crucial obstacle in developing heuristic approaches is eliminated. On the other hand the special structure of the solution induced by subadditive prices is still considered.

¿From now on we will refer to the already introduced indicator notation for bundles y_{jl} instead of the equivalent set representation $\mathcal{B}_l$ and replace constraints (2) and (3) by the following set of conditions:

$$\sum_{l\in L}\left(\theta_{il}\sum_{j\in J} r_{ij}y_{jl} - p_{il}\right) \geq \sum_{j\in J} u_{ij} + \sum_{l\in L} v_{il} + \sum_{j\in J} r_{ij} \quad i \in I \tag{9}$$

$$\sum_{j\in J} u_{ij}y_{jl} + v_{il} \geq -p_l \qquad i \in I, l \in L \tag{10}$$

$$u_{ij} \geq -r_{ij} \qquad i \in I, j \in J \tag{11}$$

$$-u_{ij} \geq 0 \qquad i \in I, j \in J \tag{12}$$

$$v_{il} \geq 0 \qquad i \in I, l \in L \tag{13}$$

where u_{ij} and v_{il} are additional variables. As is shown in [5] every optimal solution of MHM is also feasible in the new formulation NBM (New Bundling Model) described by the target function (1) together with constraints (4) - (13).

3.2 Heuristic Approach

For NBM the following heuristic approach is suggested. Starting from a given set of bundles $\tilde{L} = \{1, \ldots, m\}$ (that could also be empty) bundle augmentation is performed successively. Augmentation step k with $L := \tilde{L} \cup \{k\}$ uses fixed values y_{jl}, $j \in J, l = 1, \ldots. k-1$, for the description of bundles determined so far and $y_{jk} \in \{0, 1\}$, $j \in J$, for finding the 'optimal' bundle k. Subsequently, the heuristic approach is solved for optimal prices and the bundle candidate k. If the target function (1) can be increased the new bundle (y_{jk}^{opt}, $j \in J$) is added to the set $\tilde{L}$ and the procedure iterates, otherwise the algorithm terminates. This heuristic approach generates a sequence of monotonic increasing lower bounds for (1). It should be noticed that nonlinearities evolve from the introduction of the y_{jk}. However, this is manageable without any difficulties since only one new bundle has to be generated in each step and standard linearization techniques in integer programming can be applied. A flowchart of the heuristic approach is given in Fig. 1.

3.3 Example

Suppose, a firm intends to bundle five products ($j = 1, \ldots, 5$) and sell them to six segments ($i = 1, \ldots, 6$) of equal size (we use $N_i \equiv 1$, $i \in I$). Segment-specific reservation prices and costs for the products are given in Table 1 from which corresponding values for the bundles can easily be determined. As the complexity of such problems is mainly dependent on the number

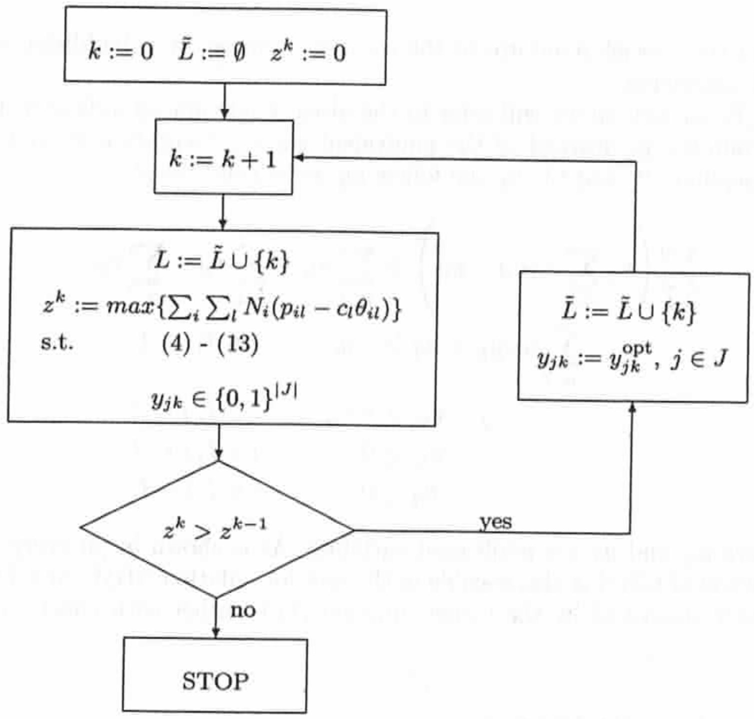

Fig. 1. Heuristic Approach for NBM

of segments considered, and the number of products used for bundling and results in considerable computational efforts even for this quite small problem, we used the proposed greedy-like heuristic approach which starts from the empty set of bundles and successively generates at each iteration step

			r_{ij}		
(i,j)	$j=1$	2	3	4	5
$i=1$	13	15	16	17	23
$i=2$	14	35	17	10	21
$i=3$	30	25	9	10	14
$i=4$	23	10	23	32	19
$i=5$	13	20	14	40	12
$i=6$	32	30	27	19	29

			c_j		
(i,j)	$j=1$	2	3	4	5
$\forall i$	29	10	15	10	25

Table 1. Problem parameter

y_{jl}	$l=1$	$l=2$	$l=3$	$l=4$	$l=5$	$l=6$	
$j=1$	0	1	0	0	1	0	
$j=2$	1	1	1	0	1	1	
$j=3$	1	0	1	1	1	0	
$j=4$	1	0	0	1	1	1	
$j=5$	0	0	0	0	1	0	
			p_l^k				z^k
$k=1$	62	-	-	-	-	-	108
$k=2$	62	55	-	-	-	-	124
$k=3$	65	55	52	-	-	-	133
$k=4$	69	55	52	55	-	-	141
$k=5$	74	55	52	55	130	-	153
$k=6$	74	55	52	55	130	60	154

Table 2. Solution tableau

k a new bundle candidate y_{jk}, $j \in J$, and bundle prices p_l^k $(l = 1, \dots, k)$. The solutions at each iteration are tracked by Table 2 that shows the bundle composition, respective prices and how the value of the target function (1) increases.[2] We further note that y_{j1}, $j \in J$, the first candidate selected in the heuristic approach, has been priced out systematically in the course of the procedure which finally leads to the rejection of this bundle by all customers. Furthermore, segment 1 does not buy at all. It is worth noting that with the proposed bundling strategy a nearly complete skimming of consumer surplus is possible. Only segment $i = 6$ would get a positive surplus from the first bundle but this surplus is too low to keep it from switching to the bundle with the highest surplus.[3]

Computation time for each iteration of the heuristic approach was within a fraction of a second.[4]

A comparison of the heuristic results and the exact solution obtained via NBM showed that both solutions were identical, i.e., both solution methods provide the same set of bundles that should be introduced. In contrary, if bundling wouldn't be taken into account an optimal pricing schedule for the product would leave the customers much more surplus resulting in a decrease of overall profit for the supplier of the - now - single products by 27%.

[2] One should mention that in cases in which different bundles would create the same surplus for a segment the heuristic approach selects that bundle with the highest profit for the bundle supplier.

[3] In the underlying problem bundle 2 and 5 yield the same surplus of 7 units, thus, the procedure decides for bundle 5 that generates the highest profit.

[4] The heuristic solution procedure has been implemented using Ilog's Cplex solver. The solver was integrated in an overall Java routine via Ilog's concert interface.

4 Conclusions

In recent years product bundling has become a prevalent marketing practice. However, only a few approaches exist that deal with the determination of optimal bundling and pricing. Promising work was done by Hanson, Martin [6]. But a drawback of their approach is that it requires considerable computational effort and that their mathematical modeling complicates the formulation of heuristic approaches. In this paper, a new approach was presented that allows the application of a simple greedy-like procedure. An example shows how easy it is to get good solutions.

References

1. Adams, W. J., Yellen, Y. L. (1976) Comodity Bundling and the Burden of Monopoly. Quarterly Journal of Economics 90, 475–498
2. Chen, Y. (1997) Equilibrium Product Bundling. Journal of Business 70, 85–103
3. Estelami, H. (1999) Consumer Savings in Complementary Product Bundles. Journal of Marketing: Theory and Practice 7, 107–114
4. Fürderer, R. (1996) Option and Component Bundling Under Demand Risk : Mass Customization Strategies in the Automobile Industry . Wiesbaden : Dt. Univ.-Verl.
5. Gaul, W., Stauß, B. (2003) Designing and Pricing new Bundles. Diskussionspapier No. 255, Institut für Entscheidungstheorie und Unternehmensforschung Universität Karlsruhe
6. Hanson, W. A., Martin, R. K. (1990) Optimal Bundle Pricing. Management Science 36, 155–174
7. Jedidi, K., Jagpal S., Manchanda P. (2003) Measuring Heterogeneous Reservation Prices for Product Bundles. Marketing Science 22, 107–130
8. Jedidi, K., Zhang, Z. J. (2002) Augmenting Conjoint Analysis to Estimate Consumer Reservation Price. Management Science 48, 1350–1368
9. Kaicker, A., Bearden, W. O., Manning, K. C. (1995) Component Versus Bundle Pricing - The Role of Selling Price Deviation from Price Expectations. Journal of Business Research 33, 231–239
10. Lewbel, A. (1985) Bundling of Substitutes or Compliments. International Journal of Industrial Organization 3, 101-107
11. Salinger, M. A. (1995) A Graphical Analysis of Bundling. Journal of Business 68, 85–98
12. Schmalensee, R. (1984) Gaussian Demand and Commodity Bundling. Journal of Business 57, 211–230
13. Stigler, G. (1963) United States vs. Loew's Inc: A Note on Block-Booking. Supreme Court Review 152, 152–157
14. Swaminathan, J. M. (2001) Enabling Customization Using Standard Operations. California Management Review 43, 125–135
15. Telser, L. G. (1979) A Theory of Monopoly of Complementary Goods. Journal of Business 52, 211–230
16. Venkatesh, R., Mahajan, V. (1993) A Probabilistic Approach to Pricing a Bundle of Products or Services. Journal of Marketing Research 30, 494-508
17. Yadav, M. S. (1994) How Buyers Evaluate Product Bundles: A Model of Anchoring and Adjustment. Journal of Customer Research 21, 342–353