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REGULAR ARTICLE

A note on consistency improvements of AHP paired comparison data

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Abstract The Analytic Hierarchy Process (AHP) is a popular multicriteria decision-making approach but the ease of AHP paired comparison data collection entails the problem that consistency restrictions have to be fulfilled for the data evaluation task. Quite a lot of consistency improvement techniques are available, however, this note explains why consistency adjustments are not necessarily helpful for computing acceptable weights for the determination of the underlying overall objective function.

Keywords Analytic Hierarchy Process · Consistency improvement · Multicriteria decision-making · Paired comparison data

Mathematics Subject Classification 15A18 · 62J15 · 68R10 · 90B50 · 91B06

1 Motivation

Analytic Hierarchy Process (AHP) is the acronym for a multicriteria decision-making approach that since its introduction (see, e.g., Saaty 1980) has attracted increasing attention (see Fig. 1 for the number of papers per year related to the query "analytic hierarchy process" in the Science Direct Database).

AHP has experienced a multitude of applications in numerous different areas such as education, engineering, management, manufacturing, and marketing to mention just a few (see, e.g., the overviews by Vaidya and Kumar 2006; Ho 2008; Ho et al.

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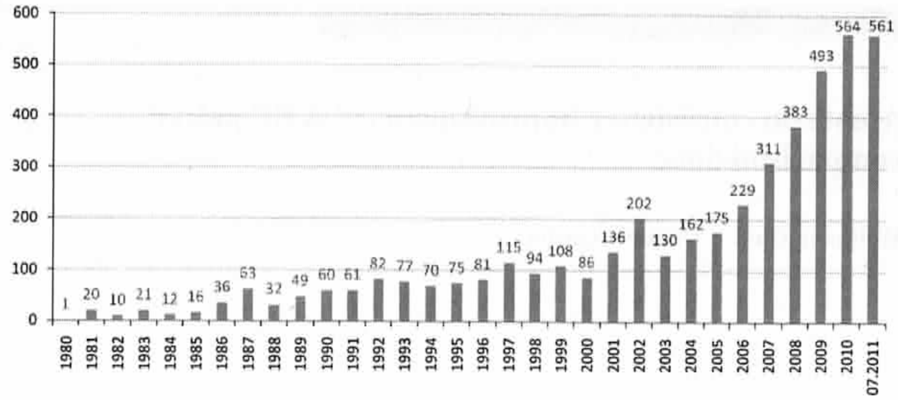

Fig. 1 Number of articles by year related to the query "analytic hierarchy process" in the Science Direct Database (July 2011)

2010). AHP can be combined with (and compared to) other methods where multicriteria decision-making activities are performed and weights or part worth utilities for salient criteria have to be determined. From a marketing point of view, e.g., preference analysis for market share predictions, product development tasks, project portfolio evaluations, quality function deployment, and SWOT (strengths, weaknesses, opportunities, threats) analysis can be mentioned as examples for multiattribute preference measurements in which AHP techniques can be applied (see, e.g., Netzer et al. 2008; Park et al. 2008; Meissner et al. 2010; Scholz et al. 2010). One reason for the popularity of AHP is probably the simple description of the approach and the ease of data collection within a judgment process via paired comparisons. However, the limited capabilities of the judging persons lead to inconsistencies with respect to constraints that have to be fulfilled in the AHP approach in order to be able to derive acceptable weights for the criteria used as ingredients for the overall objective function. As a consequence the matrix of paired comparisons reported by a judging person (abbreviated as the reported matrix) is in most cases different from the consistent, "true" matrix which would describe the person's real opinion about the decision situation (assuming that a "true" interior valuation of the judging person with respect to the underlying evaluation task exists) which by the simplified judgment process has been broken down into many paired comparisons. This problem is well known and has led to quite a number of discussions concerning consistency improvement techniques (see, e.g., Harker 1987; Dadkhah and Zahedi 1993; Zeshui and Cuiping 1999; Saaty 2003; Ishizaka and Lusti 2004; Lia and Ma 2007; Cao et al. 2010; Lin et al. 2008; Bozoki et al. 2010) by which the inconsistent reported matrix is adjusted (abbreviated as the adjusted matrix) to an acceptable level of inconsistency. Originally, we wanted to introduce a further consistency improving technique and check how favorably it compares with known counterparts. But during this research project we found an interesting result. The consistency improvement activities do not necessarily lead back to the "true" matrix. The adjusted matrix is often more similar to another consistent paired comparison matrix different from the "true" one. This situation and its implications are exposed below. Section 2 starts with basic notations, a short description of the

AHP approach, and provides reasons for the inconsistency of reported paired comparison matrices. In Sect. 3 we introduce consistency improvement possibilities and sketch some consistency improving techniques while in Sect. 4 a simulation study is described that shows to which extent the adjusted matrices are still different from the underlying consistent "true" matrices. In Sect. 5 a concluding discussion and an outlook are presented. Additionally, an appendix is added which explains a possibility for supporting a judging person so as to report a consistent paired comparison matrix.

2 Notation and AHP description

The AHP is a frequently used approach in the area of multicriteria decision-making because the underlying technique is simple, easy to understand, and straightforward to apply. Typically, a main objective of judging persons or decision-makers using the AHP is to calculate weights for hierarchically organized elements on different decision levels where the elements can be objectives, subobjectives, tangible or intangible criteria or attributes that help to describe alternatives for which a preference decision is needed. The alternatives are the basic items that one wants to value on a bottom level with the help of hierarchically ordered evaluation tasks with respect to prespecified element sets on different higher levels. Thus, a multicriteria decision problem, that can be described in this way, is divided into $H + 1$ subproblems, where each subproblem h ($h = 0, 1, \ldots, H$) contains n_h elements which are compared to each other pairwise with respect to an element of the corresponding next higher level. $h = 0$ corresponds to the evaluation task of the alternatives at the bottom level. Through comparisons between all pairs of elements of subproblem h a paired comparison matrix A_h is constructed. As further explanations do neither focus on the hierarchical structure nor on aggregation aspects with respect to different levels, the index h is omitted in the following.

Thus, the starting point for our considerations is a set of n elements that are compared pairwise with respect to an element of the next higher level resulting in a paired comparison matrix

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix} \tag{1}$$

for which the normalized eigenvector $\mathbf{w}(A) = (w_1(A), \ldots, w_n(A))^\top$ assigned to the principal eigenvalue $\lambda_{max}(A)$ of A in the eigenvalue problem

$$A\mathbf{w}(A) = \lambda_{max}(A)\mathbf{w}(A) \tag{2}$$

represents the weights of the elements that the decision-maker is looking for. In paired comparison situations the discussion about the limitations of human capacity for processing information (see, e.g., Miller 1956; Saaty and Ozdemir 2003 for arguments that a meaningful handling of between 5 and 9 alternatives seems reasonable) has to be considered and was one of the reasons why in AHP applications often the so-called Saaty scale $S = \{\frac{1}{9}, \frac{1}{8}, \ldots, \frac{1}{2}, 1, 2, \ldots, 8, 9\}$ (which only uses the numbers $1, 2, \ldots, 9$ and their reciprocals) was suggested, i.e., in paired comparisons of elements i and j a value of 1 is assigned to the verbal statement "element i and element j are equally important", e.g., a value of 3 is assigned to the judgment "element i is preferred to element j" and, e.g., a value of 9 is assigned to the verbal expression "element i is extremely preferred to (or absolutely dominant w.r.t.) element j". Intermediate values are used for more detailed assessments and reciprocals for vice versa evaluations (see Saaty 2003). Given these explanations, in the traditional AHP situation one wants to provide a paired comparison matrix A (see Eq. 1) under the assumptions

$$a_{ij} \in S = \left\{\frac{1}{9}, \frac{1}{8}, \ldots, 1, \ldots, 8, 9\right\}, \quad \forall\, i, j \quad \text{(Saaty scale)} \tag{3}$$

$$a_{ij} \cdot a_{ji} = 1, \quad \forall\, i, j \quad \text{(reciprocity)} \tag{4}$$

$$a_{ij} = a_{ik} \cdot a_{kj}, \ \forall\, i, j, k \ \text{(consistency)} \tag{5}$$

Discussions whether the Saaty scale (condition 3) is appropriate or whether other scales could be used will not be repeated (see, e.g., Jensen 1984, Ma and Zheng 1991; Donegan et al. 1992; Triantaphyllou et al. 1994; Dong et al. 2008; Liang et al. 2008; Ishizaka and Labib 2011). For the sake of simplicity we will use Saaty scale conform "true" matrices in the simulation study described in Sect. 4 (where the expression "Saaty scale conform" is used as abbreviation for solutions "conforming to the Saaty scale"), but note that in the results also paired comparisons for which the Saaty scale restriction (3) may be violated are checked. Reciprocity (condition 4) is straightforward to understand. In applications condition (5) is often violated. Thus, a consistency index (CI) was suggested to measure the degree of violation of the consistency assumption of matrix A which has the form

$$CI(A) = \frac{\lambda_{max}(A) - n}{n - 1} \tag{6}$$

In order to obtain standardized results for matrices of different size, the CI value is related to the average random consistency index $RI(n)$ of randomly chosen reciprocal matrices of size $n \times n$ constructed via entries the values of which are taken from the Saaty scale. The so-called consistency ratio (CR) of matrix A is given as

$$CR(A) = \frac{CI(A)}{RI(n)} \tag{7}$$

and A is called "fully consistent" or "zero-consistent" (or consistent for short) if $CR(A) = 0$, "α-consistent" if $0 < CR(A) \leq \alpha$, and "α-inconsistent" if $CR(A) > \alpha$ for $\alpha \in (0, 1]$. Usually, $\alpha = 0.1$ is chosen where this choice of α was motivated by statistical experiments (Vargas 1982). α-inconsistency may be due to the limitations of the Saaty scale (see also Finan and Hurley 1999; Beynon 2002) or to inaccuracies caused by a lack of concentration or uncertainties of the judging person (see, e.g., Meissner et al. 2010 for a thorough discussion concerning response errors in pairwise comparison-based preference measurements). Especially when the number of elements which have to be compared is high, it is a challenging task for the decision-maker to provide all comparisons in an α-consistent or even zero-consistent manner.

3 Consistency improvement

For a $n \times n$ paired comparison matrix A that fulfills the conditions (3), (4), and (5) one gets $a_{ij} = \frac{w_i(A)}{w_j(A)}$, $\forall\, i, j \in \{1, \ldots, n\}$, and $rang(A) = 1$, where $\lambda_{max}(A) = n$ is the only non-zero eigenvalue, i.e., the consistency ratio has the value $CR(A) = 0$. In applications paired comparison matrices are often neither consistent nor, at least, α-consistent. In order to ensure the achievement of an at least α-consistent solution, two kinds of support have been suggested.

3.1 Incorporation of the decision-maker into the adjustment process

(I) In the reassessment approach, if AFTER data collection the paired comparison matrix is not sufficiently consistent, the "most inconsistent" entry of the matrix is selected and the judging person is asked to consider and eventually reassess her/his evaluation for this entry. This procedure is repeated until the adjusted matrix is at least α-consistent.

(II) DURING the judgment process a kind of tutoring system can be used which guides the decision-maker by showing a range in which a comparison value has to lie in order to allow the overall solution to stay within a given consistency interval (see, e.g., Ishizaka and Lusti 2004).

In both cases the judgment behavior of the decision-maker is directly influenced. Disadvantages of these approaches are, e.g., that judging persons, who concentrate on evaluating elements in their imagination of consistency, may—now—answer not according to their convictions only to respond to the consistency standards that they are urged to fulfill.

3.2 Automatic adjustments

In this case a reported paired comparison matrix which is not sufficiently consistent is changed automatically and an adjusted matrix is determined without asking the judging person for her/his opinion. As many authors have tackled this case (see, e.g., Harker 1987; Zeshui and Cuiping 1999; Saaty 2003; Cao et al. 2010; Lin et al. 2008), we will briefly describe only two approaches that we use for comparison purposes—an early one (the MDA (maximum deviation algorithm) by Harker 1987, also described by Saaty 2003) and a more recent one (the exponential smoothening technique (EST) by Cao et al. 2010)—together with our new variant based on particle swarm optimization.

Denote by $T = (t_{ij})$ the "true" (normally unknown) consistent paired comparison matrix. Let $X = (x_{ij})$ be the notation for the reported matrix whereas $Y = (y_{ij})$ (if X is α-inconsistent) describes the adjusted matrix. Figure 2 depicts the situation from—what could be called—a traditional point of view.

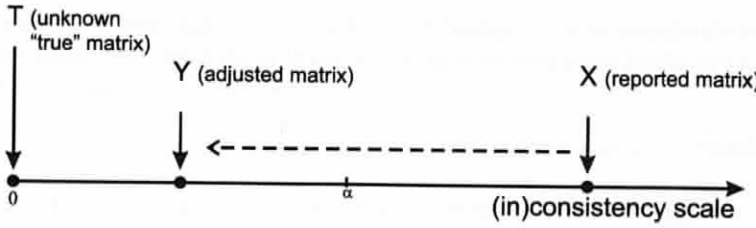

Fig. 2 Description of the consistency improvement situation from a traditional point of view (*dashed line* depicts improvement direction)

3.2.1 Maximum deviation algorithm (MDA)

For a reported α-inconsistent matrix X the entry x_{ij} for which $d_{ij} = x_{ij} \frac{w_j(X)}{w_i(X)}$ shows the largest deviation from the value 1 (note that for a consistent matrix $d_{ij} \equiv 1 \forall_{i,j}$ would be valid) is altered (together with the entry x_{ji}). Use the intermediate matrix $\tilde{X}$ which coincides with X except for $\tilde{x}_{ii} = \tilde{x}_{jj} = 2$ and $\tilde{x}_{ij} = \tilde{x}_{ji} = 0$, denote $X^{old} := X$, and calculate the altered matrix X^{new} as copy of X^{old} except for $x_{ij}^{new} = \frac{w_i(\tilde{X})}{w_j(\tilde{X})}$ (and $x_{ji}^{new} = \frac{1}{x_{ij}^{new}}$). Set $Y := X^{new}$ if X^{new} is α-consistent (note that the Saaty scale restriction (3) may be violated). The idea dates back to Harker (1987) and was used in Saaty's Expert Choice Software (Saaty 2003). This is the automated procedure of variant (I) of Sect. 3.1 which can be named MDA (maximum deviation algorithm).

3.2.2 Exponential smoothening technique (EST)

More than one entry of a reported α-inconsistent matrix X can be simultaneously altered by this approach. In iteration step k a deviation matrix $D^{(k)} = X^{(k)} \odot W^{(k)\top}$ (with $X^{(1)} = X$, $W^{(k)} = (\frac{w_i(X^{(k)})}{w_j(X^{(k)})})$, $\odot$ elementwise multiplication of matrix entries) is calculated. An intermediate matrix $\tilde{D}^{(k)} = \gamma D^{(k)} \oplus (1-\gamma) D_1$ (with $D_1 = (1)$ as $n \times n$ matrix all elements of which have value 1, $\gamma \in (0, 1]$, $\oplus$ elementwise addition of matrix entries) is used to obtain the altered matrix $X^{(k+1)} = X^{(k)} \cdot \tilde{D}^{(k)}$. Parameter γ influences how fast $X^{(k)}$ reaches α-consistency. The literature suggests a value of γ near to 1 (see, e.g., Cao et al. 2010). Set $Y := X^{(k)}$ if $X^{(k)}$ is α-consistent (note that the Saaty scale restriction (3) may be violated). This approach has been named EST (exponential smoothening technique) by the authors Cao et al. (2010).

3.2.3 Particle swarm optimization (PSO)

For a reported α-inconsistent matrix X the minimization problem

$$\min \left\{ \sum_{i,j} (x_{ij} - y_{ij})^2 \right\}$$

$$y_{ij} \in S = \left\{\frac{1}{9}, \frac{1}{8}, \ldots, 1, \ldots, 8, 9\right\} \;\forall\, i, j \;\text{(Saaty scale)} \tag{8}$$
$$y_{ij} \cdot y_{ji} = 1 \;\forall\, i, j \;\text{(reciprocity)}$$
$$CR(Y) \leq \alpha(= 0.1) \qquad (\alpha\text{-consistency})$$

computes an α-consistent Saaty scale conform solution Y for which the squared matrix error between X and Y is optimized (i.e., Y is a most similar Saaty scale conform and α-consistent solution with respect to X). For reasons that we explain in the next section we applied a particle swarm algorithm (see, e.g., Kennedy and Eberhart 1995) to the above optimization problem (8) and used the set of all $n \times n$ consistent and Saaty scale conform paired comparison matrices as starting swarm. For this approach the abbreviation particle swarm optimization (PSO) will be used.

4 Do consistency improvement techniques support the AHP?

As described in the preceding section, consistency improving techniques alter a reported α-inconsistent matrix X into an adjusted matrix Y that (perhaps, after some improvement steps) is at least α-consistent (with or without the property of Saaty scale conformity). The judging person's "true" matrix T, that (s)he—in many cases—is not able to report because of (her) his limited capabilities to match with the restrictions of the paired comparison data collection procedure, is typically unknown (see Fig. 2 for the traditional description of the consistency improvement situation). However, as we will see in this section, an acceptable consistency ratio value $CR(Y)$ of the adjusted matrix Y alone is not always a proper indicator that a good solution has been found. In fact, the weight vector $\mathbf{w}(Y)$ is the important information needed in order to determine the overall objective function of the underlying AHP situation but—as the "true" matrix T is normally unknown—it can only be hoped that $cor\ (\mathbf{w}(T), \mathbf{w}(Y))$ is sufficiently high. This was the starting point for the following Monte Carlo simulation.

4.1 Reported and underlying "true" matrices

For the construction of a reported matrix $X = (x_{ij})$ originating from a zero-consistent and Saaty scale conform "true" matrix $T = (t_{ij})$ an error matrix $E = (\epsilon_{ij})$ with $\epsilon_{ij} \sim logN(1, \sigma^2)$ and $\epsilon_{ji} = \frac{1}{\epsilon_{ij}}$ was used. As $\tilde{X} = T \odot E$ is not necessarily Saaty scale conform, the entries $\tilde{x}_{ij}$ of $\tilde{X}$ were changed to $x_{ij} = \underset{s \in S}{argmin} \left\{\left|\tilde{x_{ij}} - s\right|\right\}$ and $x_{ji} = \frac{1}{x_{ij}}$ (a procedure known from the literature, see, e.g., Crawford and Williams 1985; Carmone et al. 1997; Saaty 2003). With $T_l, l = 1, \ldots, 100$, randomly selected "true" matrices and error matrices $E_{(m,\sigma^2)}, m = 1, \ldots, 10$, and $\sigma^2 \in \{0.25, 0.5, 0.75, 1, 1.25, 1.5, 1.75, 2\}$, altogether 8,000 reported matrices $X_{(l,m,\sigma^2)}$ were constructed, for which 24,000 adjusted matrices $Y^M_{(l,m,\sigma^2)}$ were determined with the help of the described methods $M \in \{EST, MDA, PSO\}$. Using $CR(Y^M_{(l,m,\sigma^2)})$, $cor(\mathbf{w}(T_{(l)}), \mathbf{w}(Y^M_{(l,m,\sigma^2)}))$, and as additional measure

$$MSD\left(T_{(l)}, Y^M_{(l,m,\sigma^2)}\right) = \frac{1}{n \cdot (n-1)} \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} \left(t_{(l)ij} - y^M_{(l,m,\sigma^2)ij}\right)^2 \qquad (9)$$

(the Mean Squared Deviation between the matrix elements of $T_{(l)}$ and $Y^{(M)}_{(l,m,\sigma^2)}$), the following interpretation of the results is possible.

4.2 Results

The calculations on which the results are based were performed for different sizes of the underlying element sets. As the conclusions don't differ, only the results for $n = 6$ are reported. As can be expected, for increasing σ^2 the inconsistency of $X_{(l,m,\sigma^2)}$ increases (Fig. 3) and the correlation between the weights of $X_{(l,m,\sigma^2)}$ and the "true" matrix $T_{(l)}$, from which $X_{(l,m,\sigma^2)}$ originates, decreases (Fig. 4).

When the "true" matrices are unknown only measurements between the reported matrices and the adjusted matrices can be calculated (as has been done in the literature) and used for a judgment which method M produces α-consistent solutions that "best" coincide with the reported data. These results will not be shown because in this paper the more interesting question is whether the measures between the "true" matrices $T_{(l)}$ and the adjusted matrices $Y^M_{(l,m,\sigma^2)}$ indicate that the consistency improvements have been successful. As parallel ordered boxplots are too complex for depiction in a single figure, the $\left(\frac{1}{1000}\sum_{l=1}^{100}\sum_{m=1}^{10}\right)$-averaged correlations (Fig. 5) and the MSD values (Fig. 6) distinguished by $M \in$ {EST, MDA, PSO} are shown for different σ^2 together with the corresponding values when the reported matrices (X) are still not adjusted (not adj.).

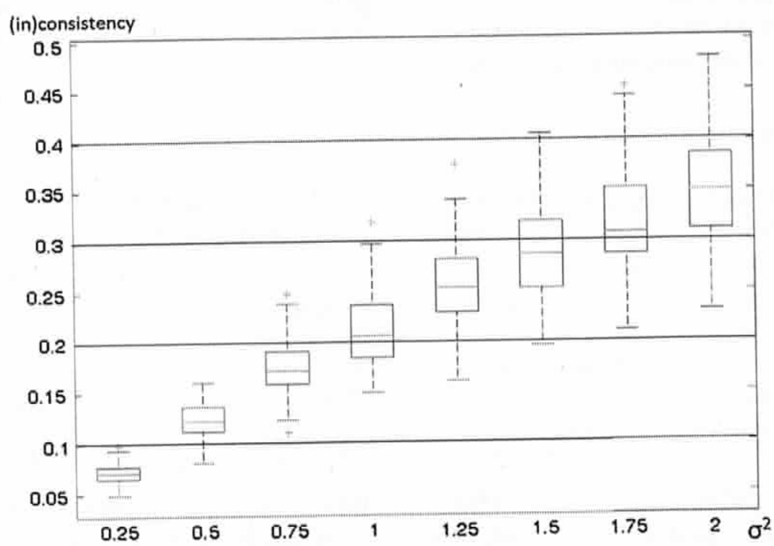

Fig. 3 Boxplots of consistency ratios $CR(X_{(l,m,\sigma^2)})$ dependent on σ^2

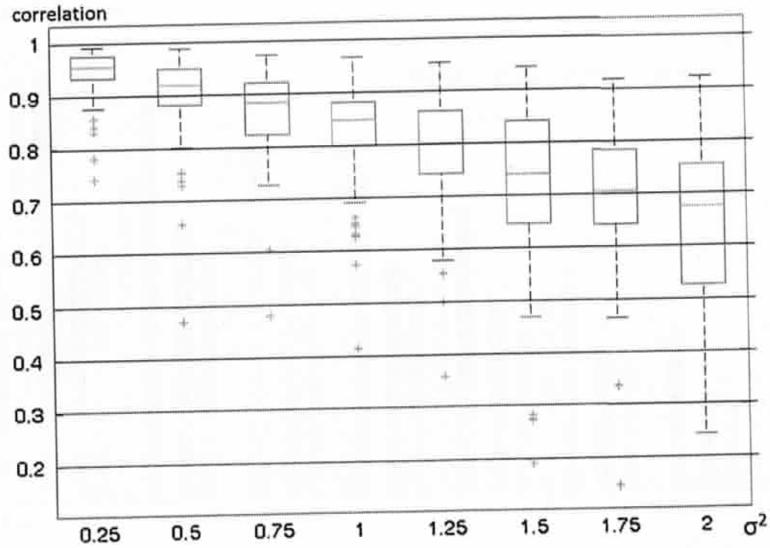

Fig. 4 Boxplots of correlations $cor(\mathbf{w}(T_{(l)}), \mathbf{w}(X_{(l,m,\sigma^2)}))$ dependent on σ^2

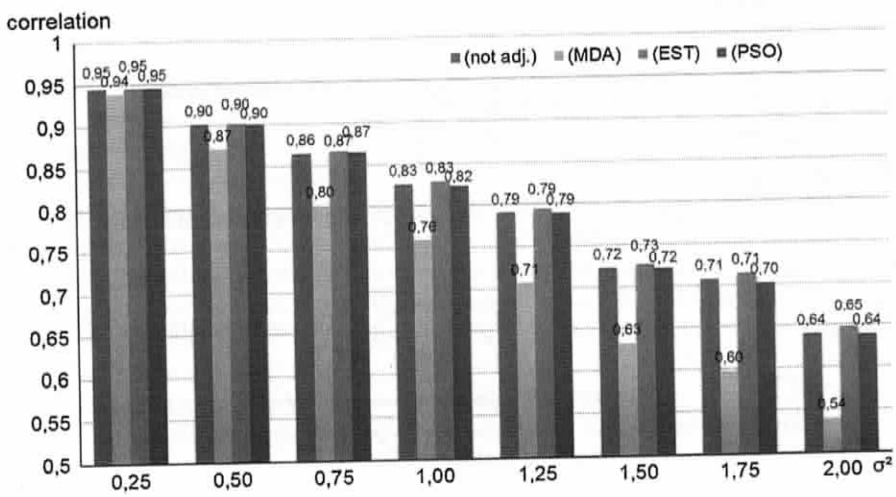

Fig. 5 Average correlations for $cor(\mathbf{w}(T), \mathbf{w}(Y^M))$ dependent on σ^2

From Fig. 5 one can conclude that on average the correlations of the weights between "true" (T) and adjusted (Y^M) matrices were nearly not improved compared to the corresponding values when the still not adjusted reported matrices are used (for MDA the corresponding values are significantly worse). For larger σ^2 [i.e. when consistency ratio values become larger (see Fig. 3)], the consistency improvement techniques fail to produce adjusted matrices the weights of which have "high" correlations with the weights of the underlying "true" matrices.

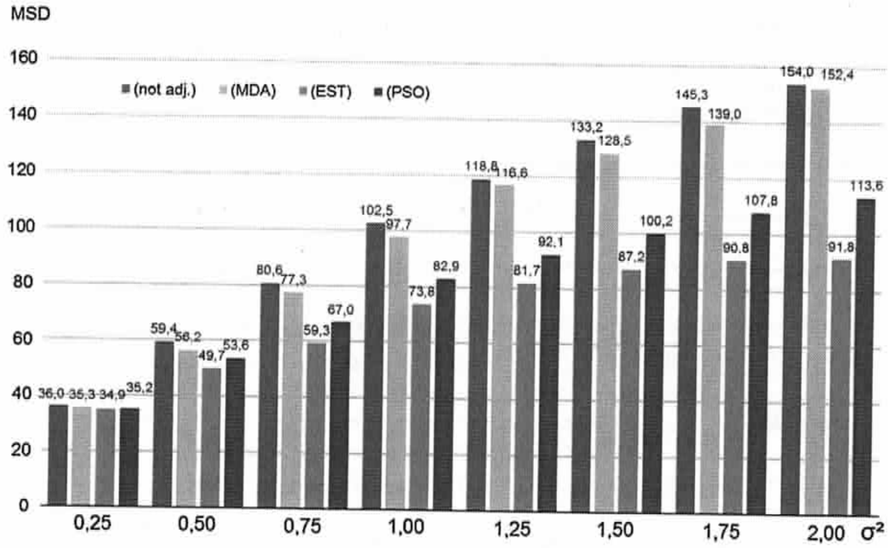

Fig. 6 Average MSD values between true matrices T and adjusted matrices Y^M dependent on σ^2

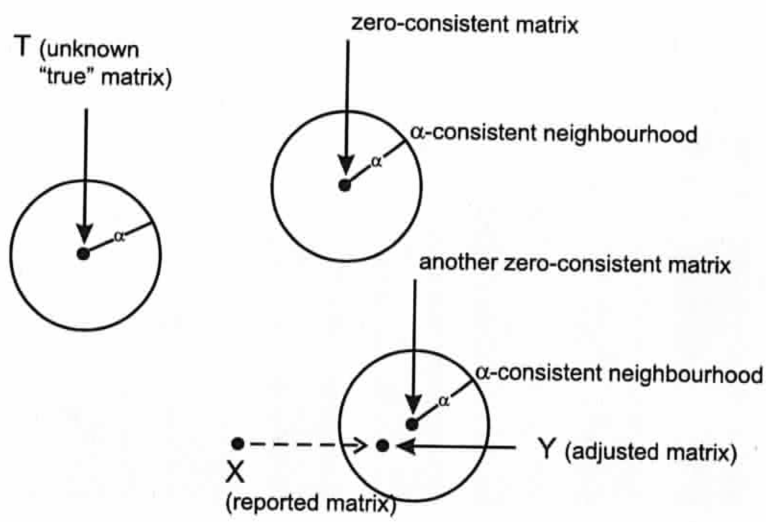

Fig. 7 Description of the consistency improvement situation from the point of view of this article (*dashed line* depicts improvement direction)

Figure 6 shows that EST and PSO find solutions which have on average smaller differences between the values of the corresponding matrix entries compared to the not adjusted and MDA solutions. Thus, one can say that they produce adjusted matrices that look more similar to the true underlying matrices.

These results allow a clear message. Consistency improvement efforts of the kind described above do not guarantee that the adjusted matrices approach the "true" matrices to a desired extent. An explanation for this phenomenon is depicted in Fig. 7.

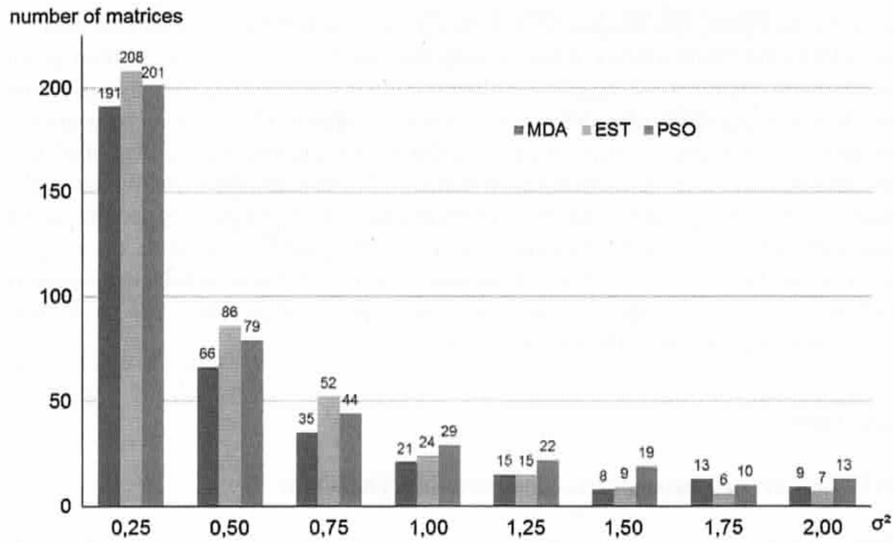

Fig. 8 Numbers $t_{\sigma^2}^M$ of correctly found underlying "true" matrices T

As there can exist large numbers of zero-consistent matrices for an underlying problem, a consistency improvement technique applied to the reported matrix X—originally originating from the "true" matrix T—can perform alterations which cause the adjusted matrix Y to enter the α-consistent neighbourhood of another zero-consistent matrix. Figure 7 also reveals why we decided to use particle swarm optimization for the solution of problem (8) for which we had initially selected a different genetic algorithm. If we use all zero-consistent and Saaty scale conform paired comparison matrices (from which our "true" matrices were randomly selected) as the starting swarm set and allow swarm elements to move within the α-consistent neighbourhoods of these zero-consistent matrices, the most-similar α-consistent Saaty scale conform (step-dependent) solution with respect to the reported matrix X will be selected in each step. From this point of view, the PSO solutions support the given interpretation of the observed phenomenon and can serve as a benchmark.

Finally, consistency improvement techniques can be applied to determine zero-consistent adjustments. Figure 8 shows the numbers $t_{\sigma^2}^M$ of correctly found "true" matrices by means of the different methods dependent on σ^2.

In other words, the results depicted in Fig. 8 reveal that in $1000 - t_{\sigma^2}^M$ cases the consistency improvement method M directed the adjusted matrices to zero-consistent matrices which did not coincide with the starting "true" matrices.

5 Conclusion and outlook

Although more than the three consistency improvement techniques used in our experiment have been suggested in the literature, approaches which attempt to adjust the consistency ratio alone seem to suffer from the same shortcoming: improving consistency does not guarantee that the adjusted matrix Y has entered such a neighbourhood of the underlying "true" matrix T (describing the judging person's real opinion about the decision situation that was broken down into "easier" paired comparisons) from which a convergence to the desired solution will happen. Correlations between the weights derived from the "true" matrix and the adjusted matrices as well as MSD values between the "true" and the adjusted matrices would help, but normally the "true" matrix is unknown. Future research will have to understand why judging persons do not state sufficiently consistent preferences and to try to support the decision situation by other means beyond consistency adjustments. One possibility to aid decision-makers with respect to their judgment task was recently proposed in Gaul (2012)[1] A short description is provided in the following appendix.

Appendix

AHP paired comparison zero-consistent data collection

Main step 0:
Ask the judging person to select a preference order for the elements under consideration: $i_1 \preceq i_2 \preceq \ldots \preceq i_n$
Main step s: $(s = 1, \ldots, n-1)$
Ask the judging person to compare the s-best element i_s with the worst element i_n: $x_{i_s i_n}$ (and $x_{i_n i_s} := \frac{1}{x_{i_s i_n}}$)
Auxiliary calculations suggest consistent paired comparison values
Auxiliary step t: $(t = 1, \ldots, s-1)$
$x_{i_t i_s} := x_{i_t i_n} \cdot \frac{1}{x_{i_s i_n}}$ (and $x_{i_s i_t} := \frac{1}{x_{i_t i_s}}$)
If the judging person is not content
Go to *main step* s^* where $1 \leq s^* \leq s$ (Recalculation from *main step* s^*)
otherwise
Go to *main step* 0 (Restart with new preference order)
otherwise
$s := s + 1$
If $s = n$ STOP
otherwise
Go to *main step s* (Continuation of data collection)

Three types of values can be distinguished in the above data collection situation: accepted values, reported values, and suggested values.
In the optimal case only $n - 1$ values reported in the data collection procedure by the judging person (reported values) are needed if all values suggested by the auxiliary calculations (suggested values) are accepted by the decision-maker (accepted values). Otherwise, a recalculation based on a new reported value $x_{i_{s^*} i_n}$ starts from main step

[1] Paper presented at the 4th Japanese-German Symposium on Classification, Kyoto, Japan, March 9–10, 2012.

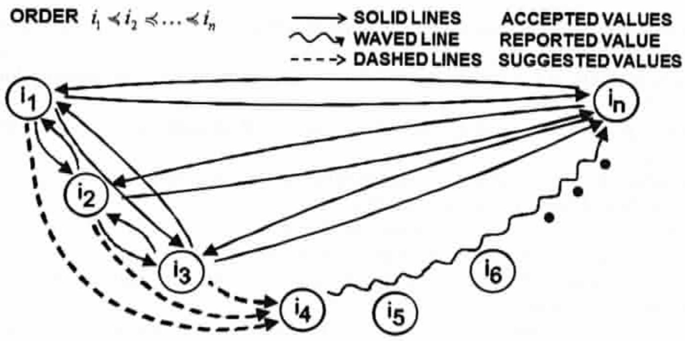

Fig. 9 Preference subgraph for main step $s = 4$ within the AHP data collection situation (arcs with reciprocal values for $s = 4$ are not depicted)

s^* ($1 \leq s^* \leq s$) or even a new preference order $i_1^{new} \preceq i_2^{new} \preceq \ldots \preceq i_n^{new}$ of the elements can be taken into consideration, in which case a restart of the procedure is performed (note that a preference order of the elements under consideration is not necessarily required (any order would suffice), but it supports the evaluation task of providing paired comparisons (monotonicity of the reported values) and even allows that the decision-maker reports Saaty scale values although the whole paired comparison matrix will not necessarily be Saaty scale conform). Figure 9 depicts the situation for main step $s = 4$ with the help of a preference subgraph.

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